133

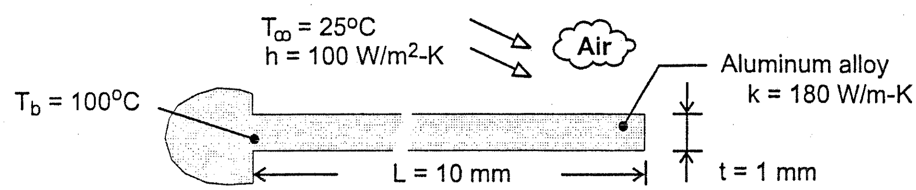
3.~~131~~

PROBLEM ~~3.132~~

KNOWN: Thickness, length, thermal conductivity, and base temperature of a rectangular fin. Fluid temperature and convection coefficient.

FIND: (a) Heat rate per unit width, efficiency, effectiveness, thermal resistance, and tip temperature for different tip conditions, (b) Effect of fin length and thermal conductivity on the heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along fin, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Fin width is much longer than thickness (w >> t). negligible

ANALYSIS: (a) The fin heat transfer rate for Cases A, B and D are given by Eqs. (3.72), (3.76) and (3.80), where $M \approx (2\,hw^2tk)^{1/2}\,(T_b - T_\infty) = (2 \times 100\ \mathrm{W/m^2{\cdot}K} \times 0.001\mathrm{m} \times 180\ \mathrm{W/m{\cdot}K})^{1/2}\,(75°\mathrm{C})\ w = 450\ w\ \mathrm{W}$, $m \approx (2h/kt)^{1/2} = (200\ \mathrm{W/m^2{\cdot}K}/180\ \mathrm{W/m{\cdot}K} \times 0.001\mathrm{m})^{1/2} = 33.3\mathrm{m^{-1}}$, $mL \approx 33.3\mathrm{m^{-1}} \times 0.010\mathrm{m} = 0.333$, and $(h/mk) \approx (100\ \mathrm{W/m^2{\cdot}K}/33.3\mathrm{m^{-1}} \times 180\ \mathrm{W/m{\cdot}K}) = 0.0167$. From Table B-1, it follows that sinh mL ≈ 0.340, cosh mL ≈ 1.057, and tanh mL ≈ 0.321. From knowledge of q_f, Eqs. (3.86), (3.81) and (3.83) yield

$A_f = 2Lw + wt + Lt$, ignore this

→ cross section of fin

$$\eta_f \approx \frac{q'_f}{h(2L+t)\theta_b},\quad \varepsilon_f \approx \frac{q'_f}{ht\,\theta_b},\quad R'_{t,f} = \frac{\theta_b}{q'_f}$$

$$\frac{A_f}{w} = 2L + t + \frac{Lt}{w}$$

Case A: From Eq. (3.72), (3.86), (3.81), (3.83) and (3.70),

3.72
$$q'_f = \frac{M}{w}\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} = 450\ \mathrm{W/m}\,\frac{0.340 + 0.0167 \times 1.057}{1.057 + 0.0167 \times 0.340} = 151\ \mathrm{W/m} \quad <$$

per unit width
$$\eta_f = \frac{151\ \mathrm{W/m}}{100\ \mathrm{W/m^2 \cdot K}\,(0.021\mathrm{m})\,75°\mathrm{C}} = 0.96 \quad <$$

$$\varepsilon_f = \frac{151\ \mathrm{W/m}}{100\ \mathrm{W/m^2 \cdot K}\,(0.001\mathrm{m})\,75°\mathrm{C}} = 20.1 \quad \boxed{R'_{t,f} = \frac{75°\mathrm{C}}{151\ \mathrm{W/m}} = 0.50\ \mathrm{m \cdot K/W}} \quad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL + (h/mk)\sinh mL} = 25°\mathrm{C} + \frac{75°\mathrm{C}}{1.057 + (0.0167)0.340} = 95.6°\mathrm{C} \quad <$$

Case B: From Eqs. (3.76), (3.86), (3.81), (3.83) and (3.75)

~~3.75~~
$$q'_f = \frac{M}{w}\tanh mL = 450\ \mathrm{W/m}\,(0.321) = 144\ \mathrm{W/m} \quad <$$

$$\eta_f = 0.92,\ \varepsilon_f = 19.2,\ R'_{t,f} = 0.52\ \mathrm{m \cdot K/W} \quad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL} = 25°\mathrm{C} + \frac{75°\mathrm{C}}{1.057} = 96.0°\mathrm{C} \quad <$$

Continued …..

3.133

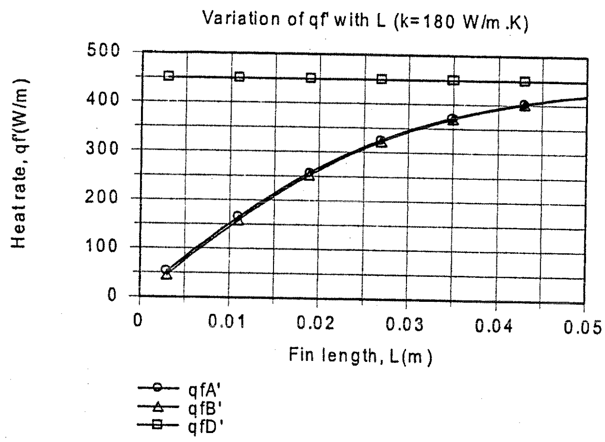
PROBLEM ~~3.122~~ (Cont.)

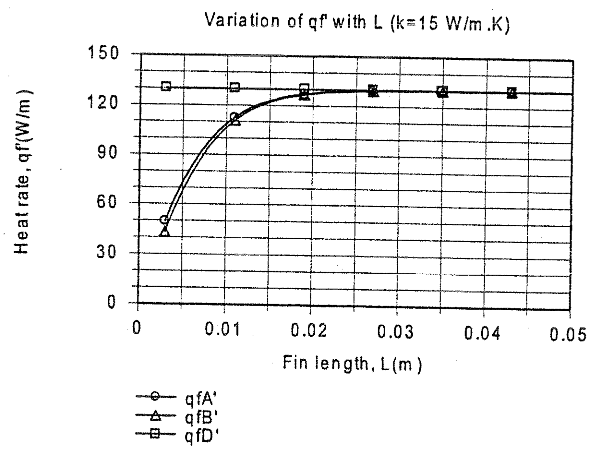
Case D ($L \rightarrow \infty$): From Eqs. (3.80), (3.86), (3.81), (3.83) and (3.79)

$$q'_f = \frac{M}{w} = 450\ \mathrm{W/m}$$ <

$$\eta_f = 0,\ \varepsilon_f = 60.0,\ R'_{t,f} = 0.167\ \mathrm{m \cdot K/W},\ T(L) = T_\infty = 25°\mathrm{C}$$ <

(b) The effect of L on the heat rate is shown below for the aluminum and stainless steel fins.





For both materials, differences between the Case A and B results diminish with increasing L and are within 1% of each other at $L \approx 27$ mm and $L \approx 13$ mm for the aluminum and steel, respectively. At L = 3 mm, results differ by 14% and 13% for the aluminum and steel, respectively. The Case A and B results approach those of the infinite fin approximation more quickly for stainless steel due to the larger temperature gradients, $|dT/dx|$, for the smaller value of k.

COMMENTS: From the results of Part (a), we see there is a slight reduction in performance (smaller values of q'_f, η_f and ε_f, as well as a larger value of $R'_{t,f}$) associated with insulating the tip. Although $\eta_f = 0$ for the infinite fin, q'_f and ε_f are substantially larger than results for L = 10 mm, indicating that performance may be significantly improved by increasing L.