ME-318 Engineering Heat Transfer

Ch. 2 Learning Objectives

What you should learn:

- How to write a mathematical expression for the heat flow rate via conduction using Fourier's Law
- How to derive the heat diffusion equation by performing an energy balance on a differential material element
- To identify boundary conditions necessary to solve the heat diffusion equation
- How to recognize the heat diffusion equation in cylindrical & spherical coordinates

Important Concepts & Equations

- Temperature Gradient:

$$\nabla T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}$$

- Heat Diffusion Equation:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}$$

- Thermal Diffusivity:

$$\alpha = \frac{k}{\rho c_p}$$

- Heat Diffusion Equation in Cylindrical Coordinates:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(k\frac{\partial T}{\partial \varphi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}$$

- Heat Diffusion Equation in Spherical Coordinates:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \varphi}\left(k\frac{\partial T}{\partial \varphi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q}$$

Adapted from V. Prantil