1.33 (a, b)

(a) \( x(t) = 3[\alpha(t+2) - \alpha(t-2)] \)

(b) \( x(t) = 2[\alpha(t) - \alpha(t-2)] \)

\[ E = \lim_{T \to \infty} \frac{1}{2} \int_{-T}^{T} x^2(t) \, dt = \lim_{T \to \infty} \frac{1}{2} \int_{-T}^{T} \alpha^2(t) \, dt = 8 < \infty \]

Power is finite.

1.35 (a) \( x(t) = \left[1 - e^{-2t}\right] \alpha(t) \).

Plots diverge to \( \infty \), so \( E \to \infty \).

Check if it's

1.38 (b) \( x(t) = e^{-at} \), \( a > 0 \).

\( x(t) \) approaches 0 exponentially.

\[ E = \lim_{T \to \infty} \frac{1}{2} \int_{-T}^{T} \alpha(t)^2 e^{-2at} \, dt = \frac{1}{a} \alpha(0) \]

2.11(b) (c)

\( y(t) = 3\sin(t) \cdot x(t) \).

Linear, but not TI.

TI? Let \( x(t) = x(t-k) \).

\[ y_1(t) = 3\sin(t) \cdot x(t-k) \] is x, into system delayed.

\[ y_2(t) = 3\sin(t-k) = y_1(t-k) \] delay output.

\[ y_1(t) = y_2(t) \] for any \( k \).

No! Since \( \sin(t) \neq \sin(t-k) \) for any \( k \).
2.1 (e) $y(t) = \frac{1}{2} \int_0^t x(t) \, dt$ \hspace{1cm} \textit{Linear + TI.}$

Can Prove as in 6.

$c(t) = x(t) + x(t)$

Interchange w/ integration

$\therefore$ integration is \textit{linear}$

$\uparrow$

TI: Delay input:

$y(t) = \int_{t-k}^{t} x(t) \, dt$

$\sigma = t - k \quad d\sigma = dt$

$\uparrow$

dummy var. Sub $t - k$

of integration - in definite

2.2 (c)

(a) $y(t) = 3x(t-1)$ is LTI. Can prove as above

(b) $y(t) = x(t)u(t)$ is L but not TI.

TI? If input is delayed, shape of output changes.

E.g. let $x(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$

Apply

$\text{system}$

$\Rightarrow$

These are not time-shifted replicas. They are truncated differently.

2.5 (a, c) $s(t) = r(t) - 2u(t-1) + u(t-2)$

$\uparrow$

\textit{Step response} short cut: Express $x(t)$ in terms of $u(t)$.

$\text{System changes each } u \text{ to } s$, just as it changes $s \to u$!

That is definition of \textit{step/impulse response}$

(a) $x(t) = c(t) - c(t-2)$ \quad $y_1(t) = s(t) - s(t-2) = (r(t) - 2u(t-1) + u(t-2))$

$+ (r(t-2) - 2u(t-2))$ - \text{sum part 5, 5, 5, 5, 5}$

(c) $x(t) = u(t) - 2u(t-1) + u(t-2) \Rightarrow y_2(t) = s(t) - 2s(t-1) + s(t-2)$

$\uparrow$

$\begin{cases} 1 & \text{for } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$

$\sum$
Determine if each of the following signals is a power signal, an energy signal, or neither.

b. \( x_2(t) = 3[u(t - 2) - u(t + 2)] \)

This is the negative of part (a), so it is still an energy signal.

```matlab
\texttt{t = linspace(-5, 5);}
\texttt{x2 = 3 * ((t>2) - (t>-2));}
\texttt{plot(t, x2)}
\texttt{xlabel('Time (s)'), ylabel('Signal level')}
\texttt{ylim([min(x2)-0.1, max(x2)+0.1])] \% so flat areas don't hit plot border}
```

We can see that all of the non-zero values of the signal are captured in the variable, so we can calculate the energy as the integral of the magnitude squared of the signal. For a real signal, the square is the same as the magnitude squared. We expect this to be 4 \( s \times 3^2 = 36 \).

```matlab
\texttt{dt = diff(t(1:2));}
\texttt{E = sum(dt * x2.^2);}
\texttt{fprintf('The energy is \%g', E)}
```

The energy is 36.3636.

This is off by about 1% from the true value, which is expected since we're making an
approximation with a finite number of time values.

\begin{verbatim}
N = length(t);
fprintf('%g values have been calculated in the function.\n', N)
\end{verbatim}

100 values have been calculated in the function.

You can pass a 3rd argument to specify how many values to calculate instead of the default of 100.
Determine if each of the following signals is a power signal, an energy signal, or neither.

a. $x_1(t) = [1 - e^{-2t}]u(t)$

This function is a causal exponential decay towards the DC value of 1. Therefore it has infinite energy but is a power signal. As we consider an infinite time window about $t=0$, in the limit, the signal is 0 for half of the time and 1 for half of the time; the exponential decay region is vanishingly small around $0^+$. So, we expect the power to be $(0^2+1^2)/2 = 1/2$.

```matlab
T = 10; % goes to infinity in power definition
t = linspace(-T/2, T/2, 1000);
x1 = (1-exp(-2*t)).*(t>0);
plot(t, x1)
xlabel('Time (s)')
ylabel('x_1(t)')
```

```matlab
dt = diff(t(1:2));
P = (1/T) * sum(dt*x1.^2); % Riemann integral
fprintf('Using %g samples, the power estimate is %g.\n', length(t), P)
```

Using 1000 samples, the power estimate is 0.425505.
The estimate is a bit low since the transient after t=0 is included, but power is a long term average. Increase T for a better estimate.