1. (2 points) Let \( f_1 = 2000 \) Hz, \( f_2 = 0 \) Hz, \( f_3 = 500 \) Hz, and \( f_4 = 1500 \) Hz. Calculate the digital frequencies, \( \omega_n \), for each frequency, \( f_n \), for \( f_3 \) through \( f_4 \). Recall that the digital frequency is how many radians a sinusoid moves through between samples. For example, if a signal is sampled 10 times per period, its digital frequency is \( 2\pi/10 \). Do not make any adjustments for aliasing.

\[
\begin{align*}
\omega_1 &= \frac{f_1}{f_s} \cdot 2\pi = \frac{2000}{2000} \cdot 2\pi = 0 \text{ radians/sample} \\
\omega_2 &= \frac{f_2}{f_s} \cdot 2\pi = \frac{0}{2000} \cdot 2\pi = 0 \\
\omega_3 &= \frac{f_3}{f_s} \cdot 2\pi = \frac{500}{2000} \cdot 2\pi = \frac{\pi}{2} \\
\omega_4 &= \frac{f_4}{f_s} \cdot 2\pi = \frac{1500}{2000} \cdot 2\pi = \frac{3\pi}{2}
\end{align*}
\]

2. (2 points) Explain whether any of the 3 sinusoids above are aliased. For each frequency that is aliased, assuming it was not stopped by a suitable antialias filter, calculate what frequency would be observed at the output of the system due to aliasing.

Digital signals are periodic in frequency, so \( \omega_3 = \omega_4 + 2\pi k \) gives some samples for all integers \( k \) \( (k \in \mathbb{Z}) \). Choose \( k = -1 \).

\( \omega_3 = \frac{3\pi}{2} - 2\pi = \frac{\pi}{2} \). Now, \( |\omega_3| \leq \pi \) so it is the non-aliased frequency.

Since signal is real, the negative freq. component also exists \( \omega_3 + \pi/2 \).

3. (4 points) Calculate the first 4 samples of the unit step response of \( y(n) = 0.5 y(n-1) - 0.5 y(n-2) \) when \( x(n) = u(n) \).

\[
\begin{align*}
y(0) &= \frac{1}{2} y(-1) + x(-1) - 3 y(-1) + 4 x(-2) \\
y(0) &= \frac{1}{2} \cdot 1 + 0 + 1 - 3 \cdot 1 + 0 = 0 \\
y(1) &= \frac{1}{2} y(0) + 1 - 3 \cdot 1 + 0 = -1.5 \\
y(2) &= \frac{1}{2} y(1) + 1 - 3 \cdot 1 + 4 \cdot 1 = 1.25
\end{align*}
\]

4. (2 points) What is the vector of "a" or autoregressive or IIR (infinite impulse response) coefficients in the above equation? (Recall that the "b" or FIR coefficients correspond to a weighted sum of inputs.)

\[
a = [1 \quad -0.5] = [a_0 \quad a_1]
\]