1. (1 point) Let \( x_1(n) = 0.5 \cos((\pi/2) n) + \cos(\pi n) \). Calculate \( X_1(e^{j\omega}) \). Recall that the DTFT of \( \cos(\omega_0 n) \) is \( \pi(\delta(\omega - \omega_0) + \delta(\omega + \omega_0)) \).

   Applying linearity:
   \[
   X_1(e^{j\omega}) = \pi \left( 0.5 (\delta(\omega - \frac{\pi}{2}) + \delta(\omega + \frac{\pi}{2})) + (\delta(\omega - \pi) + \delta(\omega + \pi)) \right)
   \]

2. (1 point) Let \( x_3(n) = x_1(n) (u(n+2) - u(n-2)) \). Calculate the samples of \( x_3(n) \).

   \[
   \begin{array}{cccccccccc}
   n & \cdots & -3 & -2 & -1 & 0 & 1 & 2 & \cdots \\
   u(n+2) & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
   u(n-2) & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
   \end{array}
   \]

   Effectively, \( n = -2:1 \):

   \[
   \begin{array}{cccccccccc}
   n & \cdots & -2 & -1 & 0 & 1 & 2 & \cdots \\
   \cos(\omega) & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\
   \end{array}
   \]

3. (1 point) Calculate \( X_2(e^{j\omega}) \) based on your answer to the previous question. Recall that \( X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \). Note: your answer will look a lot different than your answer to the first question since the sin waves are truncated in time.

   \[
   X(e^{j\omega}) = 0.5 e^{j\omega^2} - e^{j\omega} + 1.5 - e^{-j\omega}
   \]
Given the difference equation \( y(n) = 0.8 y(n-1) - 0.6 x(n) \)

4. (2 points) Take the DTFT of both sides of the equation. Recall that the DTFT of a delayed signal, \( y(n-k) \), is \( e^{-j\omega k} Y(e^{j\omega}) \).
   
   \[
   Y(e^{j\omega}) = 0.8 e^{-j\omega} Y(e^{j\omega}) - 0.6 X(e^{j\omega})
   \]

5. (2 points) Solve the above equation for transfer function \( H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \)
   
   \[
   Y(e^{j\omega})(1 - 0.8 e^{-j\omega}) = -0.6 X(e^{j\omega})
   
   H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{-0.6}{1 - 0.8 e^{-j\omega}}
   \]

6. (1 point) Let \( f_1 = 1200 \text{ Hz}, f_1 = 300 \text{ Hz}, \) and \( f_2 = 600 \text{ Hz}. \) Calculate the digital frequencies, \( \omega_n \), for each frequency, \( \omega_n \), for \( f_1 \) through \( f_2 \). Recall that the digital frequency is how many radians a sinusoid moves through between samples. For example, if a signal is sampled 10 times per period, its digital frequency is \( 2\pi/10 \).
   
   \[
   \omega_1 = \frac{f_1}{f_s} \cdot 2\pi = \frac{1200}{1500} \cdot 2\pi = \frac{24\pi}{5} \]
   
   \[
   \omega_2 = \frac{f_2}{f_s} \cdot 2\pi = \frac{600}{1500} \cdot 2\pi = \frac{2\pi}{5}
   \]

7. (2 points) Evaluate \( H \) at the digital frequencies calculated above.
   
   \[
   H(e^{j\omega}) = \frac{-0.6}{1 - 0.8 e^{-j\omega}} = \frac{-0.6}{1 - 0.8e^{-j\omega}}
   
   = \frac{0.48 - 0.6}{1.64} = -0.365 + j0.292
   
   = 0.468 \angle 24.66^\circ \text{ or useful form}
   
   = 0.468 \angle 0.785 \pi \text{ for angle}
   
   = 0.468 \angle 141.3^\circ
   \]

8. (1 point) What do these values of \( H \) tell you about the steady state response to sinusoids?
   
   The magnitude and angle of \( H \) are the gain and phase shift of a sinusoid at that frequency.