Reminder: The DTFT is defined by $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$. (Sometimes the DTFT is symbolized by $X(e^{j\omega})$ as a shorthand notation emphasizing that the function $X$ is often generalized to be defined anywhere in the complex plane, $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$. In this case the DTFT is found by evaluating $X$ at points on the unit circle, $1 \leq \omega$, that is, letting $z = e^{-j\omega}$.)

1. (2 points) Let $f_1 = 1000$ Hz, $f_1 = 0$ Hz, $f_2 = 200$ Hz, and $f_3 = 700$ Hz. Calculate the digital frequencies, $\omega_n$, for each frequency, $f_n$, for $f_1$ through $f_3$. Recall that the digital frequency is how many radians a sinusoid moves through between samples. For example, if a signal is sampled 10 times per period, its digital frequency is $2\pi/10$. Do not make any adjustments for aliasing.

2. (2 points) Explain whether any of the 3 sinusoids above are aliased. For each frequency that is aliased, assuming it was not stopped by a suitable antialias filter, calculate what frequency in hertz would be observed at the output of the system due to aliasing.
3. (1 point) Let \( x_1(n) = 2\cos((\pi/2) n) \). Calculate \( X_1(e^{j\omega}) \). Recall that the DTFT of \( \cos(\omega_0 n) \) is \( \pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)) \).

4. (1 point) Let \( x_2(n) = x_1(n) (u(n+2) - u(n-2)) \). Calculate the samples of \( x_2(n) \).

5. (1 point) Calculate \( X_2(e^{j\omega}) \) based on your answer to the previous question. Note: your answer will look a lot different than the other DTFT you calculated.

6. (1 point) Explain the following property of the DTFT: \( X(e^{j(\omega+2\pi)}) = X(e^{j\omega}) \).

7. (2 points) How is the value of the DTFT at \(-\omega\) related to its value at \(\omega\)? Assume \( x(n) \) is a real signal. (You may just state the answer if you remember it from the book, but you can also derive it from the DTFT definition; the first step is evaluating \( X(\omega) \) at \(-\omega\).)