Given the difference equation \( y(n) = 0.5 \, y(n-1) + 5 \, x(n) - 2 \, x(n-1) \)

1. (2 points) Take the z-transform of both sides of the equation. Remember, \( z^{-1} \) represents a sample delay.
   \[
   Y(z) = \frac{1}{2} \, z^{-1} \, Y(z) + 5 \, X(z) - 2 \, z^{-1} \, X(z)
   \]

2. (2 points) Solve the above equation for the transfer function \( H(z) \).
   \[
   Y(z) \left( 1 - \frac{1}{2} \, z^{-1} \right) = X(z) \left( 5 - 2 \, z^{-1} \right)
   \]
   \[
   H(z) = \frac{Y(z)}{X(z)} = \frac{5 - 2 \, z^{-1}}{1 - \frac{1}{2} \, z^{-1}} = \frac{10 \, z^{-2}}{2 - z^{-1}}
   \]
   Preferred form

3. (2 points) Let the input \( x(n) \) be the causal sequence \([1 \, 1/8 \, 1/16 \, ...]\). Note that this is a geometric series with ratio \( +1/2 \). Calculate \( X(z) \).
   \[
   X(z) = \frac{z}{z - 1/2} = \frac{1 - \frac{1}{2} \, z^{-1}}{z^{-1}} \quad \text{Preferred form}
   \]

4. (1 point) Calculate \( Y(z) \) based on \( H(z) \) and \( X(z) \) above. You DO NOT need to simplify it using partial fractions.
   \[
   Y(z) = H(z) \, X(z) = \frac{5 \, z - 2}{z - \frac{1}{2}} \cdot \frac{z}{z - \frac{1}{2}} = \frac{5z^2 - 2z}{(z - \frac{1}{2})^2}
   \]

5. (1 point) Calculate the z-transform of \( x = [6 \, -5 \, 2] \), which starts at \( n=-2 \).
   \[
   X(z) = 6z^2 - 5z + 2
   \]

6. (2 points) Calculate the inverse z-transform of \( X(z) = \frac{z^2 - 3z}{z - 1} - \frac{z - 2}{z - 0.1} \).
   \[
   x(n) = u(n) - 0.1^{n-2} \, u(n-2)
   \]