1. (3 points) The pole of a notch filter serves to cancel the zero at nearby frequencies. Depending on specifications, notch filter pole radii are typically between 0.9 and 0.995. Discuss what happens when the pole radius is
   a. Too small (e.g., 0.7)
      The notch is excessively wide.
   b. Too large (e.g., 0.999999)
      (also true that Q nonliqerent is greater)
      The notch is too narrow
      or the transient response is too long
      OR round-off cancell (unlikely, but could happen d.e. an precision)
   c. Greater than 1
      The filter has an unstable response

2. (1 point) Describe the phase response of a symmetric FIR filter. Be complete for full credit.
   It is linear and represents a constant delay of order \( T/2 \) samples.
   OR
   It is fully described by the term \( e^{-j\omega T/2} \), where \( M \) is filter length.

3. (1 point) Describe the constraints on the zero locations in a symmetric FIR filter.
   Their reciprocals must also be zeros.

   NOTE: Real coefficients further require that the conjugate zeros exist, forming constellations of 4 zeros in the general case.
   “\( z \) must be on unit circle/10/conj.” \( \Leftrightarrow \) this is true for all real \( \rightarrow \) real system.
Recall that the formula for the inverse DFT is $x(n) = \frac{1}{N} \sum_{k=1}^{N-1} w_{N}^{kn} X(k)$, where $w_{N} = e^{-j\frac{2\pi}{N}}$

4. (2 points) Calculate the $4\times4$ IDFT matrix, recalling that $n$ varies across rows and $k$ varies across columns. Express values in rectangular form.

$$N=4$$
$$w_{4}=e^{-\frac{2\pi}{4}} = -1$$

$$D^{-1} = \frac{1}{4} \begin{bmatrix} w_{4}^{0} & w_{4}^{0} & w_{4}^{0} & w_{4}^{0} \\ w_{4}^{1} & w_{4}^{1} & w_{4}^{1} & w_{4}^{1} \\ w_{4}^{2} & w_{4}^{2} & w_{4}^{2} & w_{4}^{2} \\ w_{4}^{3} & w_{4}^{3} & w_{4}^{3} & w_{4}^{3} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1+j & -1-j & 1-j & -1+j \\ 1-j & -1+j & 1+j & -1-j \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

5. (2 points) Apply that $4\times4$ matrix operator to the column vector $X(k) = [24; 4-4j; 0; 4+4j]$ to find $x(n)$, the IDFT of $X(k)$.

$$x(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1+j & -1-j & 1-j & -1+j \\ 1-j & -1+j & 1+j & -1-j \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 24 \\ 4-4j \\ 0 \\ 4+4j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 24+4+4 & 24+4+4 & 24-4+4 & 24-4+4 \\ 4+4j & 4-4j & 4+4j & 4-4j \\ 24 & 24 & 24 & 24 \\ 4+4j & 4-4j & 4+4j & 4-4j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 32 & 32 & 16 & 16 \\ -8 & 8 & 4 & -4 \end{bmatrix}$$

6. (1 point) What constraints are there on $X(k)$ when $x(n)$ is real? Be complete and unambiguous for full credit.

- $X(k)$ is conjugate symmetric (sufficient answer)
- $X(-k) = X(N-k) = X^*(k)$ (preferred answer)

Best answer shows that $x(0) \in \mathbb{R}$ and if $N/4$ even, $x(k) \in \mathbb{R}$
1. (3 points) The pole of a notch filter serves to cancel the zero at nearby frequencies. Depending on specifications, notch filter pole radii are typically between 0.9 and 0.995. Discuss what happens when the pole radius is
   a. Too small (e.g., 0.7)
   The notch is too wide.
   b. Too large (e.g., 0.999999)
   The notch is too narrow.
   OR The transient response is too long.
   (OK answers: - round-off cancellation - greater non-linearity)
   c. Greater than 1
   The filter has an unstable response.

2. (1 point) Describe the phase response of a symmetric FIR filter. Be complete for full credit.
   It is linear, reflecting a constant delay of $\frac{1}{2}$ sample.
   OR $\frac{3}{4}$
   It is fully described by the term $e^{-\frac{1}{M-1}}$, where $M$ is the filter length.

3. (1 point) Describe the constraints on the zero locations in a symmetric FIR filter.
   Then reciprocals must also be zeros.
   More: Real coefficients further require that the conjugate zeros lie on a circle and form a constellation of 4 zeros in the general case.
   (0) if first cos. (note: benzene unit circle is not required)
Recall that the formula for the DFT is $X(k) = \sum_{n=1}^{N-1} w_N^{kn} x(n)$, where $w_N = e^{-j\frac{2\pi}{N}}$.

4. (2 points) Calculate the $4\times4$ DFT matrix, recalling that $n$ varies across rows and $k$ varies across columns. Express values in rectangular form.

\[
N=4, \quad w_N = e^{-j\frac{2\pi}{4}} = -j
\]

\[
D = \begin{bmatrix}
w_0^0 & w_0^1 & w_0^2 & w_0^3 \\
w_1^0 & w_1^1 & w_1^2 & w_1^3 \\
w_2^0 & w_2^1 & w_2^2 & w_2^3 \\
w_3^0 & w_3^1 & w_3^2 & w_3^3
\end{bmatrix} = \begin{bmatrix}
 1 & 1 & 1 & 1 \\
 1 & -j & -1 & -j \\
 1 & -1 & -1 & -1 \\
 1 & j & -1 & -j
\end{bmatrix}
\]

5. (2 points) Apply that $4\times4$ matrix operator to the column vector $x(n) = [1; 2; 3; 4]$ to find $X(k)$, the DFT of $x(n)$.

\[
X(k) = D \cdot x(n) = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -j & -1 & -j \\
1 & -1 & -1 & -1 \\
1 & j & -1 & -j
\end{bmatrix} \cdot \begin{bmatrix}
1 \\
2 \\
3 \\
4
\end{bmatrix} = \begin{bmatrix}
1+2+7+4 \\
1-2j-3+4j \\
1-2+3-4 \\
1+2j-3-4j
\end{bmatrix} = \begin{bmatrix}
10 \\
2+2j \\
2 \\
2-2j
\end{bmatrix}
\]

6. (1 point) What constraints are there on $X(k)$ when $x(n)$ is real? Be complete and unambiguous for full credit.

- $X(k)$ is conjugate symmetric. *sufficient answer*
- $X(-k) = X^*(N-k)$ *preferred answer*

Best answer adds: $X(0) \in \mathbb{R}$

*If $N$ is even, $X(\frac{N}{2}) \in \mathbb{R}$*