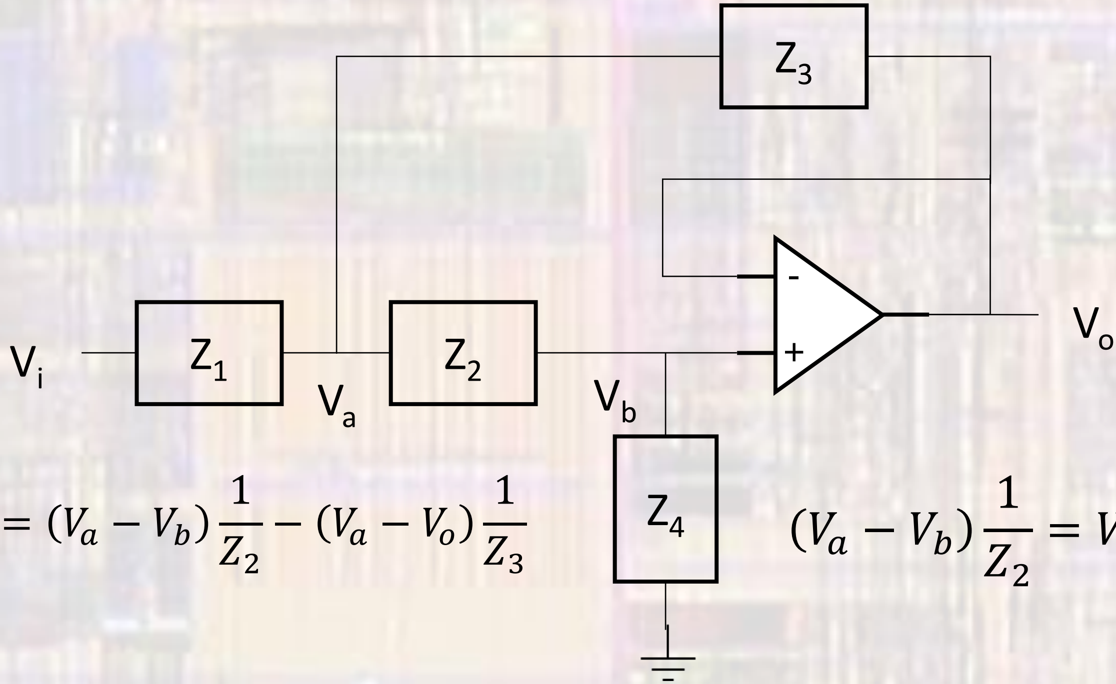
RC Active Filters
2^{nd} Order +

Last updated 4/29/22

RC Active Filters

- Generalized Second Order Configuration



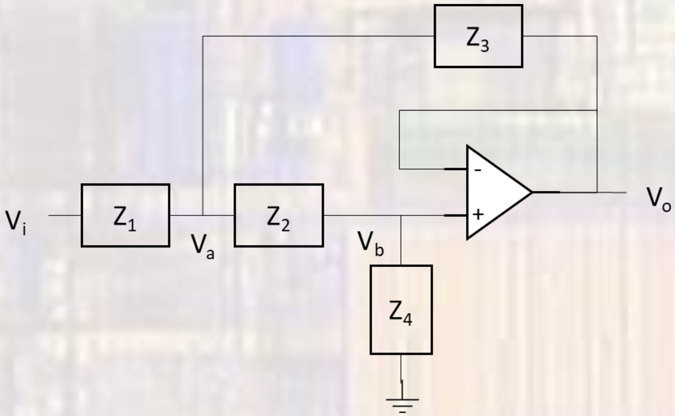
$$(V_i - V_a)\frac{1}{Z_1} = (V_a - V_b)\frac{1}{Z_2} - (V_a - V_o)\frac{1}{Z_3}$$

$$(V_a - V_b)\frac{1}{Z_2} = V_b\frac{1}{Z_4}$$

$$\boxed{A_v = \frac{\frac{1}{Z_1}\frac{1}{Z_2}}{\frac{1}{Z_1}\frac{1}{Z_2} + \frac{1}{Z_4}\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)}}$$

RC Active Filters

- Generalized Second Order Configuration



$$A_v = \frac{\frac{1}{Z_1}\frac{1}{Z_2}}{\frac{1}{Z_1}\frac{1}{Z_2} + \frac{1}{Z_4}\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)}$$

- Can be generalized to the form

$$H_s = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

ω – characteristic frequency
Q – quality factor

$a_o \neq 0$ low pass
$a_1 \neq 0$ band pass
$a_2 \neq 0$ high pass

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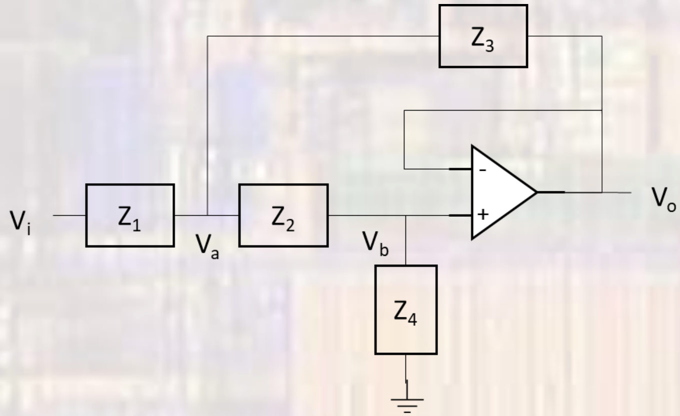
RC Active Filters

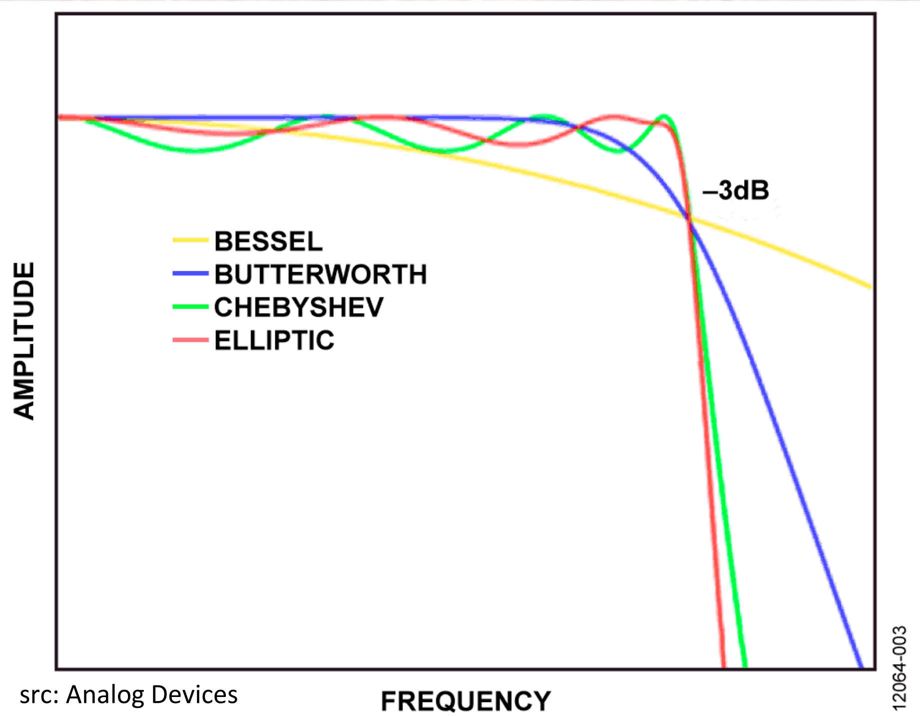
- Generalized Second Order Configuration



$$H_s = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

- Variations in a, ω, and Q generate different passband characteristics

- a, ω, and Q are set by the values of Z_1, Z_2, Z_3, Z_4



src: Analog Devices

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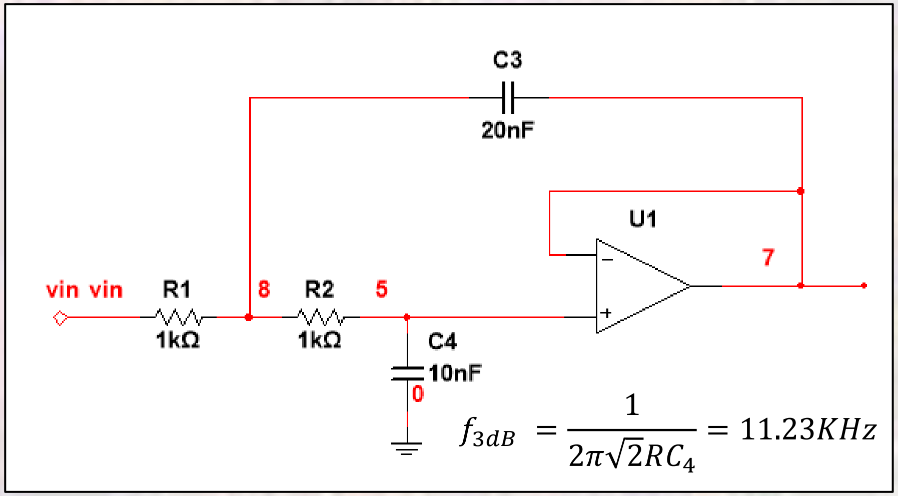
RC Active Filters

- Butterworth – Maximally Flat
  - Low Pass

$$H_s = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



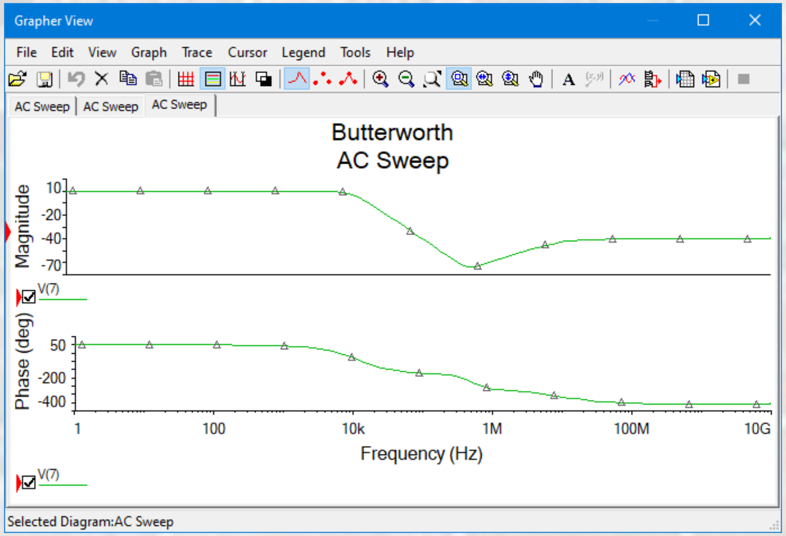
$$f_{3dB} = \frac{1}{2\pi\sqrt{2}RC_4} = 11.23KHz$$

For the Butterworth response

$R_1 = R_2$

$C_3 = 2C_4$

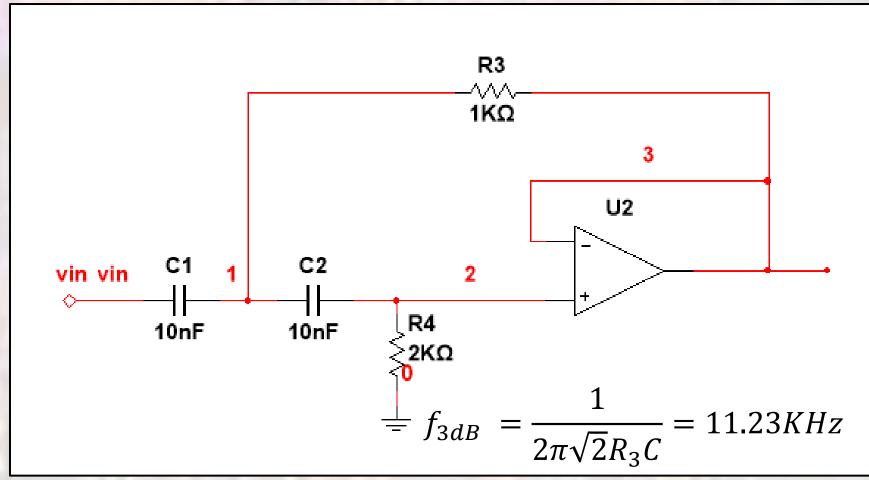
$$\omega_{3dB} = \frac{1}{\sqrt{2}RC_4}$$



RC Active Filters

- Butterworth – Maximally Flat
  - High Pass

$$H_s = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

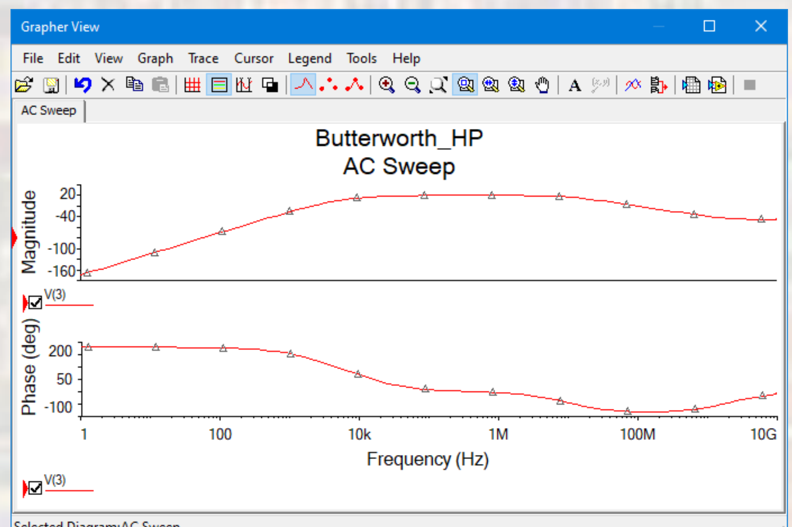


For the Butterworth response

$R_4 = 2R_3$

$C_1 = C_2$

$$\omega_{3dB} = \frac{1}{\sqrt{2}R_3C}$$



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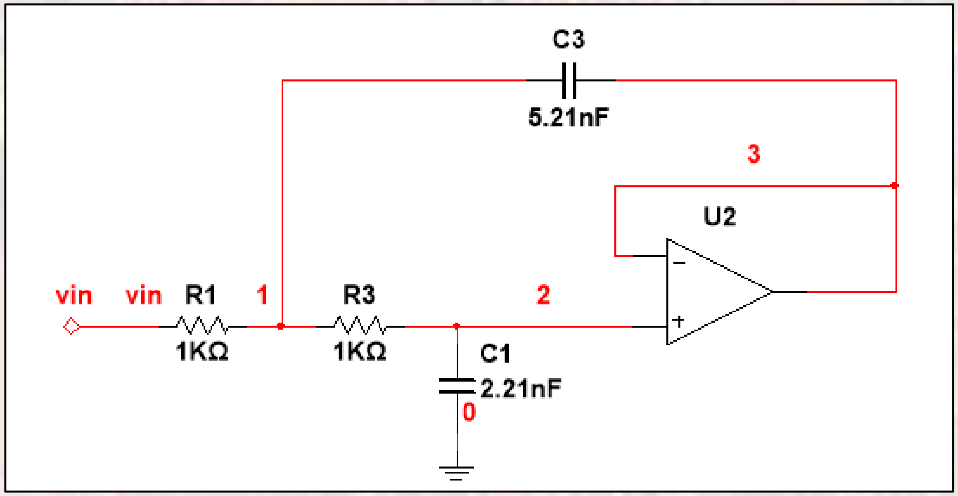
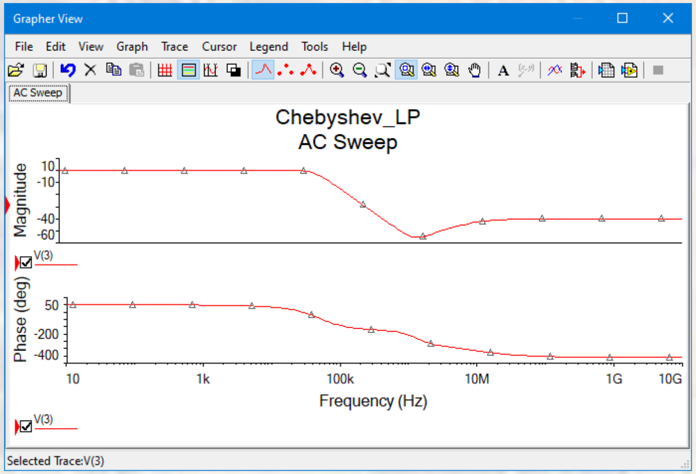
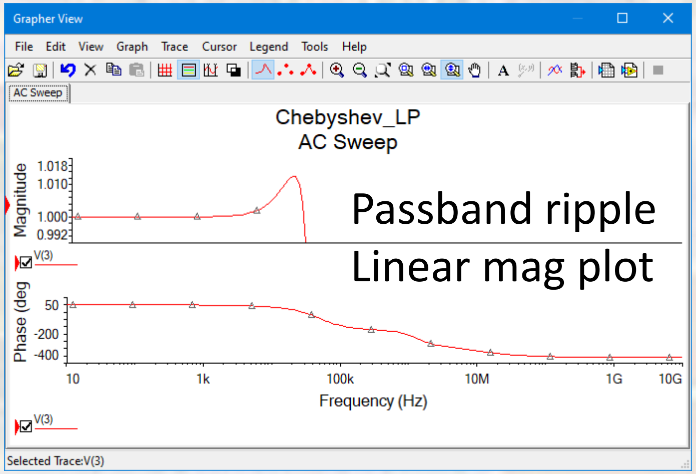
RC Active Filters

- Chebyshev – Enhanced initial rolloff
  - Low Pass

$$H_s = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

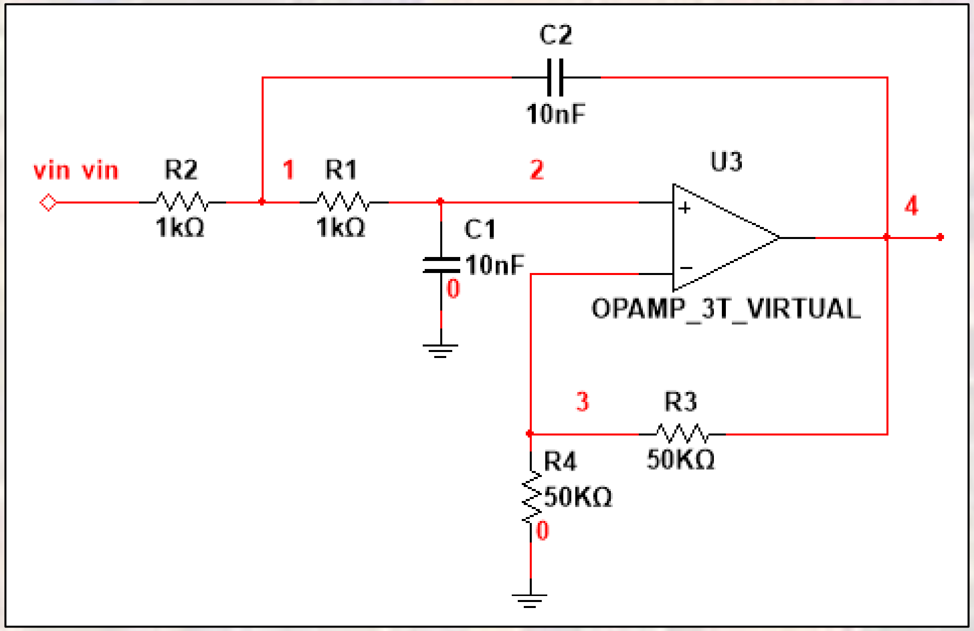






RC Active Filters

- Second order filters
  - Non-inverting
  - Selectable gain
  - High frequencies → unity gain



$$f_C = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}}$$

$$A = 1 + \frac{R_F}{R_I}$$

$$A_v = A \times \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega_0}{Q} \times \omega\right)^2}}$$

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RC Active Filters

- Second order filters
  - 2nd order filters with gain exhibit a more complex response than 2nd order passive filters
    - Q – quality factor = gain at the 3db point
    - ζ – zeta – damping factor = 1/(2Q)
    - ω_0 – characteristic frequency – where things change in the frequency response

2nd order LPF

$$A_v = A \times \left| \frac{\omega_0^2}{-\omega^2 + j\left(\frac{\omega_0}{Q}\right)\omega + \omega_0^2} \right| = A \times \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega_0}{Q} \times \omega\right)^2}}$$

$$\omega_C = \omega_0 \sqrt{1 - \frac{1}{2Q^2} + \sqrt{1 + \left(1 - \frac{1}{2Q^2}\right)^2}}$$

$$Q = \frac{1}{A - 3}$$

$$A = 3 - \frac{1}{Q}$$

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RC Active Filters

- Second order filters
  - 2nd order filters with gain exhibit a more complex response than 2nd order passive filters

2nd order LPF

$$A_v = A \times \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega_0}{Q} \times \omega\right)^2}}$$

$$\omega_C = \omega_0 \sqrt{1 - \frac{1}{2Q^2} + \sqrt{1 + \left(1 - \frac{1}{2Q^2}\right)^2}}$$

$$\omega = 0, \qquad A_v = A$$

$$\omega = \omega_0, \qquad A_v = A \times Q$$

$$\omega \gg \omega_0, \qquad A_v = A \times \frac{\omega_0^2}{\omega^2} = 0$$

$$Q = \frac{1}{\sqrt{2}}, \qquad \omega_C = \omega_0, \qquad A = 1.58$$

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RC Active Filters

- Second order filters
  - 2nd order filters with gain exhibit a more complex response than 2nd order passive filters

- Over damped: Filter response is smooth and slow

  Transient response is smooth and slow

  $A < 1.586 \qquad Q < \frac{1}{\sqrt{2}} \qquad \zeta > 0.707$

- Critically damped: Filter response matches the passive filter

  $A = 1.586 \qquad Q = \frac{1}{\sqrt{2}} \qquad \zeta = 0.707$

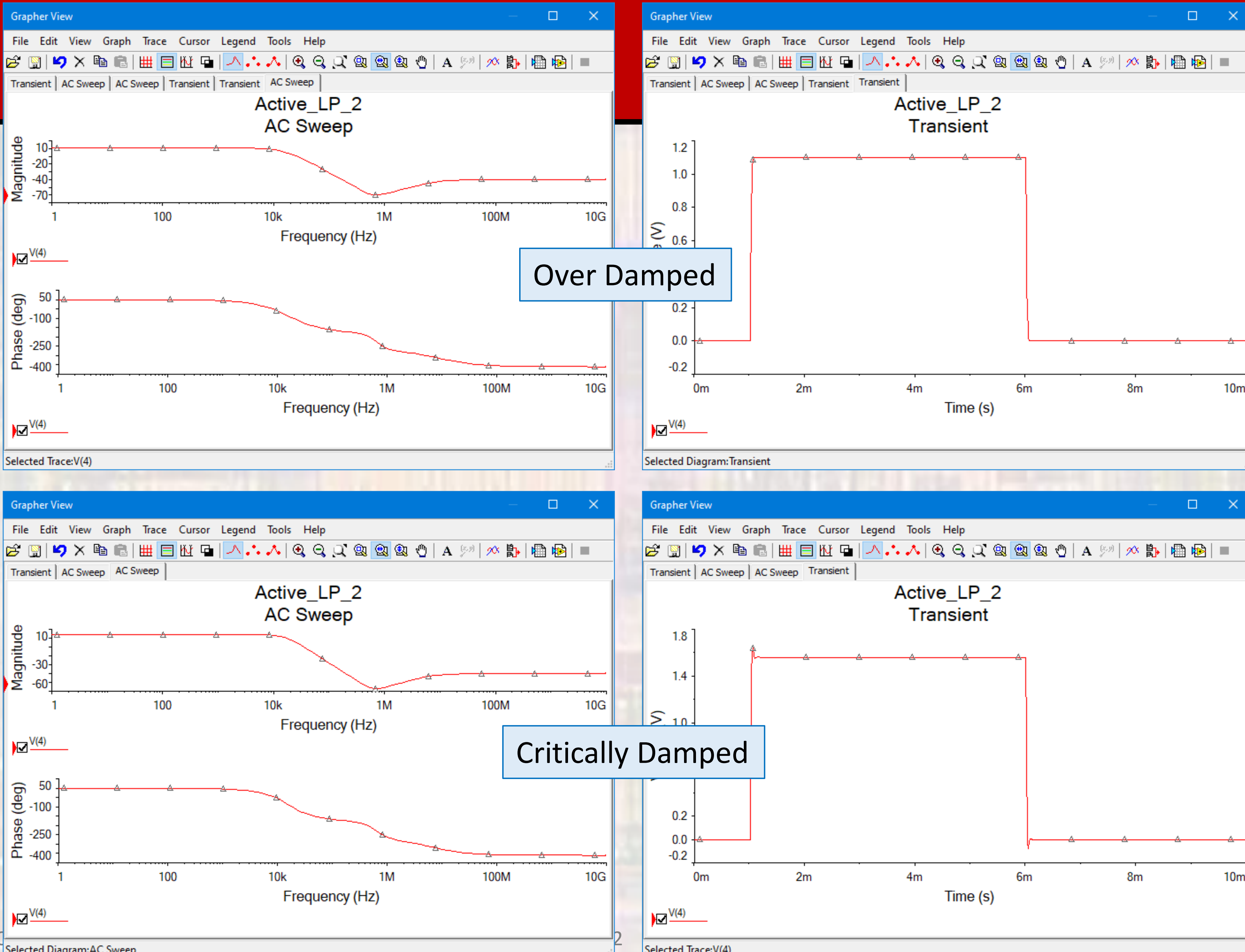
- Under damped: Filter response is peaked

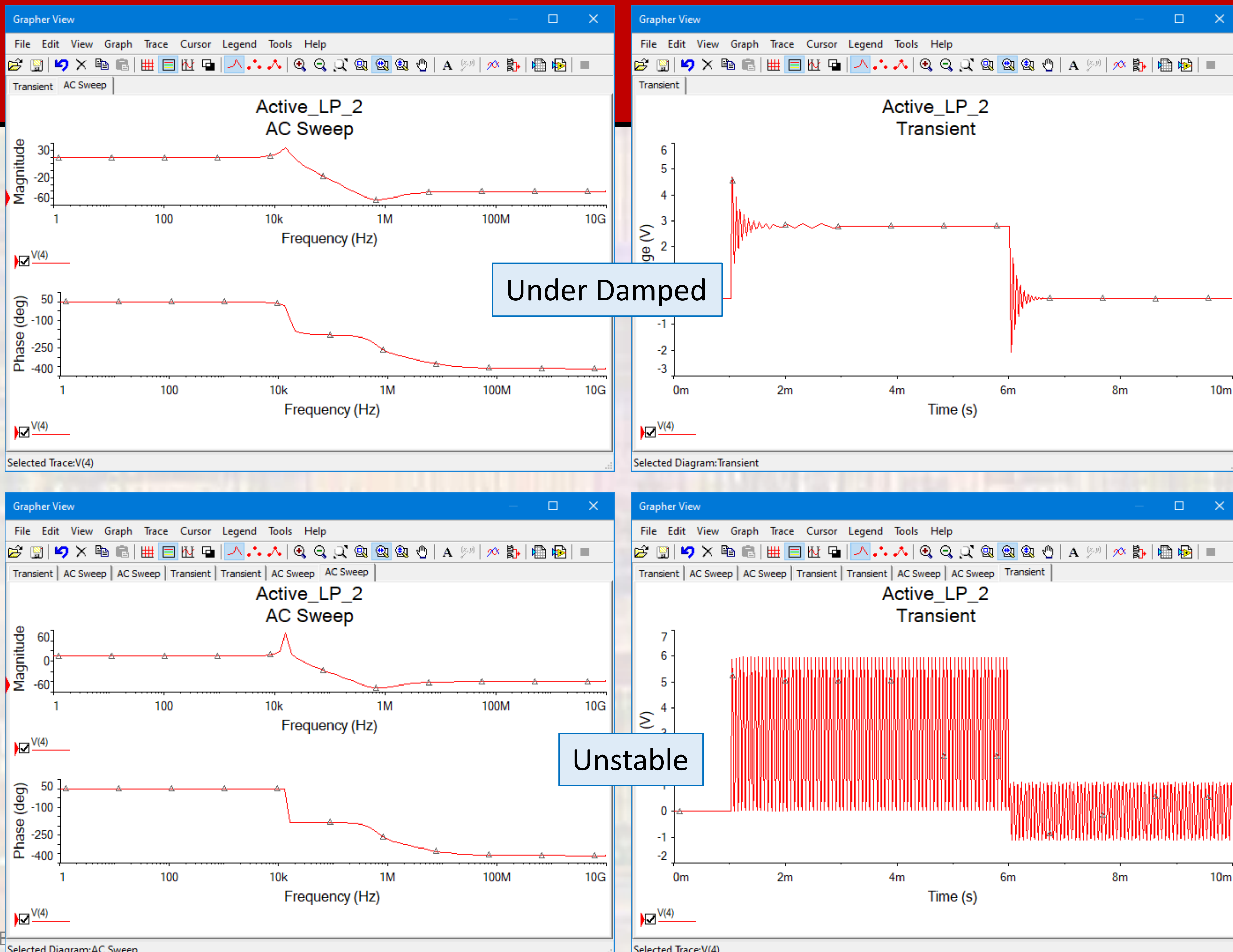
  Transient response "rings"

  $A > 1.586 \qquad Q > \frac{1}{\sqrt{2}} \qquad \zeta < 0.707$

- Unstable: Filter circuit oscillates independently

  $A \rightarrow 3 \qquad Q = \infty \qquad \zeta \rightarrow 0$


Active_LP_2
AC Sweep
Magnitude
Phase (deg)
Frequency (Hz)
V(4)
Active_LP_2
Transient
Time (s)
V(4)
Over Damped
Critically Damped
Selected Trace:V(4)
Selected Diagram:Transient
Selected Diagram:AC Sweep


Grapher View
Active_LP_2
AC Sweep
Magnitude
Phase (deg)
Frequency (Hz)
V(4)
Selected Trace:V(4)
Active_LP_2
Transient
Time (s)
V(4)
Selected Diagram:Transient
Under Damped
Active_LP_2
AC Sweep
Selected Diagram:AC Sweep
Active_LP_2
Transient
Time (s)
Selected Trace:V(4)
Unstable

RC Active Filters

- Order > 2 active Filters
  - Cascade 1st and 2nd order stages
  - Remember, gain can be tricky in 2nd order stages
    - Move gain to 1st order stages where possible

  - Computer programs and calculators available

RC Active Filters

- Chebyshev – 5^{th} order HP
  - High Pass