Binary Subtraction

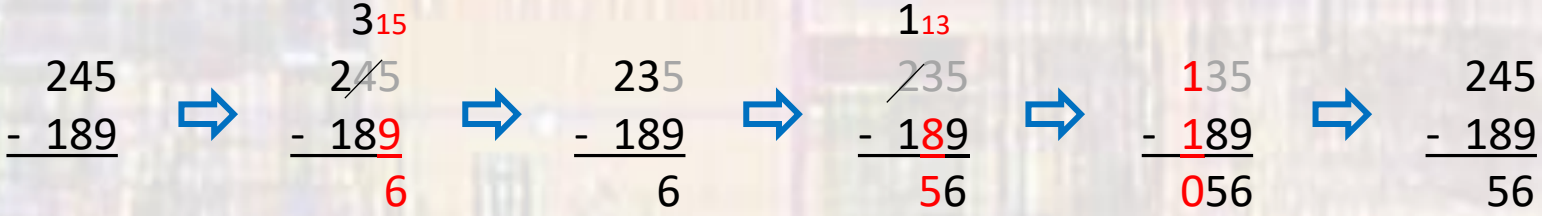
Last updated 10/3/24

These slides introduce subtraction with Binary Numbers

Binary Subtraction

- Elementary school concepts
  - Subtract columns of numbers and keep track of how much is borrowed from the next column

```
  245   minuend
- 189   subtrahend
-----
   56   difference
```



- This is very difficult to implement in hardware

Binary Subtraction

- Binary Subtraction
  - Negate the subtrahend and add
    - a – b → a + (-b)
  - Works for Signed and Unsigned binary numbers
  - Overflow – Unsigned
    - No carryout of the addition indicates overflow
    - This is the opposite of normal unsigned addition
  - Overflow – Signed
    - MSB carry-in ≠ MSB carry-out indicates overflow
    - This is the same as normal signed addition

Binary Subtraction

- Binary Subtraction
  - Unsigned

$$\begin{array}{r} 71 \\ -\ 29 \end{array} \Rightarrow \begin{array}{r} 71 \\ +\ -(29) \end{array} \Rightarrow \begin{array}{r} 71 \\ +\ (-29) \end{array}$$

$$\begin{array}{rcccccccc} & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ - & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array} \qquad \begin{array}{rr} & 71 \\ - & 29 \end{array}$$

$$\begin{array}{rcccccccc} \boxed{1} & 1 & & & & 1 & 1 & 1 & \\ & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ + & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ \hline & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{array} \qquad \begin{array}{rr} & 71 \\ + & (-29) \\ \hline & 42 \end{array}$$

Carry-out → no overflow
Answer is correct

Binary Subtraction

- Binary Subtraction
  - Unsigned

71 - 82 ⇨ 71 + -(82) ⇨ 71 + (-82)

```
  0 1 0 0 0 1 1 1        71
- 0 1 0 1 0 0 1 0    -   82
```

```
0   0       1 1 1
  0 1 0 0 0 1 1 1        71
+ 1 0 1 0 1 1 1 0    +  (-82)
  1 1 1 1 0 1 0 1       245
```

No Carry-out → overflow
Answer is incorrect

Binary Subtraction

- Unsigned Overflow – Interpretation

8 bit unsigned binary

	0	1	0	0	0	1	1	1
-	0	1	0	1	0	0	1	0

71 - 82

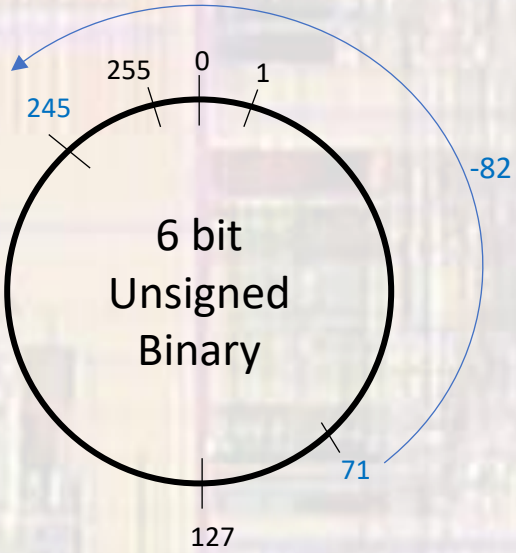
0	0			1	1	1		
	0	1	0	0	0	1	1	1
+	1	0	1	0	1	1	1	0
	1	1	1	1	0	1	0	1

71 + (-82) = 245

Overflow

No carryout

8 bit
Unsigned Binary
Number Line



Binary Subtraction

- Binary Subtraction
  - Signed

$$\begin{array}{r} -67 \\ -\ -82 \end{array} \Rightarrow \begin{array}{r} -67 \\ +\ -(-82) \end{array} \Rightarrow \begin{array}{r} -67 \\ +\ (82) \end{array}$$

$$\begin{array}{rcccccccc} & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ - & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ \hline \end{array} \qquad \begin{array}{r} -67 \\ -\ -82 \\ \hline \end{array}$$

$$\begin{array}{rcccccccc} \boxed{1} & 1 & 1 & 1 & & & & & \\ & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ + & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ \hline & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array} \qquad \begin{array}{r} -67 \\ +\ 82 \\ \hline 15 \end{array}$$

Carry-out = carry-in → no overflow
Answer is correct

Binary Subtraction

- Binary Subtraction
  - Signed

-67	-67	-67
- 82	+ -(82)	+ (-82)

	1	**0**	**1**	**1**	**1**	**1**	**0**	**1**		**-67**
-	**0**	**1**	**0**	**1**	**0**	**0**	**1**	**0**	-	**82**

1	0	1	1	1	1					
	1	**0**	**1**	**1**	**1**	**1**	**0**	**1**		**-67**
+	**1**	**0**	**1**	**0**	**1**	**1**	**1**	**0**	+	**82**
	0	**1**	**1**	**0**	**1**	**0**	**1**	**1**		**117**

Carry-out ≠ carry-in → overflow
Answer is incorrect

Binary Subtraction

- Signed Overflow – Interpretation

8 bit signed binary

	1	0	1	1	1	1	0	1
-	0	1	0	1	0	0	1	0

-67 - 82

1	0	1	1	1	1			
	1	0	1	1	1	1	0	1
+	1	0	1	0	1	1	1	0
	0	1	1	0	1	0	1	1

-67 + 82 = 117

Overflow

MSB Carry-out ≠ carry-in

8 bit Signed Binary Number Line