Boolean Logic

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Boolean Logic

- Boolean logic, Boolean Algebra
  - A topic of mathematics that studies logic
  - Formalized by (credited to) George Boole (1847)
  - Includes:
    - Abstraction of T/F to 1/0
    - Set of basic operations
    - Set of terms
    - Rules to manipulate equations

Boolean Logic

- Abstraction and basic operands
  - Abstraction
    - T → 1
    - F → 0
  - Logic Expression
    - Operation Operand → 1 or 0
    - Operand Operation Operand → 1 or 0
  - Operations
    - NOT – flips the evaluation of the operand
      - 1 → 0 or 0 → 1
    - OR – evaluates as 1 if either operand is 1
    - AND – evaluates as 1 if both operands are 1

Boolean Logic

- NOT – flips the evaluation of the operand
  - 1 → 0 or 0 → 1
  - not operand
  - ~ operand
  - $\overline{\text{operand}}$

A = 1
B = 0

not A → 0
~ B → 1

not (not A) → 1

$\overline{\sim A}$ → 1

NOT

A	$\overline{A}$
0	1
1	0

Boolean Logic

- OR – evaluates as 1(T) if either input is 1 (T)
  - op1 or op2
  - op1 | op1
  - op1 + op2

A = 1
B = 0
C = 1

A or B → 1
A | C → 1
B + C → 1

(not A) | B → 0

OR

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

Boolean Logic

- AND – evaluates as 1(T) if both inputs are 1(T)
  - op1 and op2
  - op1 & op2
  - op1op2, (op1)(op2)

A = 1
B = 0
C = 1

A and B → 0
A & C → 1
BC → 0

A & (not B) → 1

AND

A	B	A B
0	0	0
0	1	0
1	0	0
1	1	1

Boolean Logic

- Boolean Logic Precedence
  - NOT >> AND >> OR

$AB + \overline{C} \rightarrow ((AB) + (\overline{C}))$

Boolean Logic

- Terms
  - Complement
    - The NOT of a variable
      - The complement of A is $\overline{A}$
      - The complement of $\overline{A}$ is A
  - Literal
    - Any single variable or it's complement
    - A, $\overline{B}$, $\overline{C}$, D
  - Product (implicant)
    - The AND of 2 or more literals
    - AB, $\overline{A}B\overline{C}$
  - Sum
    - The OR of 2 or more literals
    - $\overline{A}$+B, A+$\overline{B}$+C

Boolean Logic

- Terms – cont'd
  - Minterm
    - The logical combination of all input variables to make a row in the truth table true (1)
    - Labeled m_x , where x is the row in the truth table
      - Starting with row 0 at the top

A	B	Y	minterm	minterm name
0	0		$\overline{A}\,\overline{B}$	m_0
0	1		$\overline{A}\,B$	m_1
1	0		$A\,\overline{B}$	m_2
1	1		$A\,B$	m_3

Boolean Logic

- Terms – cont'd
  - Sum of Products
    - The sum of all minterms that result in a 1 for the output
      - $Z = \overline{A}\,B + A\,\overline{B}$
      - $Z = \sum(m_1, m_3) = \sum(1,3)$

	A	B	Z	minterm	minterm name
	0	0	0	$\overline{A}\,\overline{B}$	m_0
→	0	1	1	$\overline{A}\,B$	m_1
	1	0	0	$A\,\overline{B}$	m_2
→	1	1	1	$A\,B$	m_3

Boolean Logic

- Terms – cont'd
  - Maxterm
    - The logical combination of all input variables to make a row in the truth table false (0)
    - Labeled M_x, where x is the row in the truth table
      - Starting with row 0 at the top

A	B	X	maxterm	maxterm name
0	0		$A+B$	M_0
0	1		$A+\overline{B}$	M_1
1	0		$\overline{A}+B$	M_2
1	1		$\overline{A}+\overline{B}$	M_3

Boolean Logic

- Terms – cont'd
  - Product of Sums Form (POS)
    - The product of all maxterms that result in a 0 for the output
      - X = (A+$\overline{B}$) ($\overline{A}$+B)
      - $X = \prod(M_1, M_2) = \prod(1, 2)$

	A	B	X	maxterm	maxterm name
	0	0	1	A+B	M_0
→	0	1	0	A+$\overline{B}$	M_1
→	1	0	0	$\overline{A}$+B	M_2
	1	1	1	$\overline{A}$+$\overline{B}$	M_3