Boolean Logic Formal Laws

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These slides show formal laws for Boolean logic

Boolean Logic – Formal Laws

- Logic expression laws / identities

Reminder – this is not algebra
+ → OR
* → AND

#	Theorem	Dual	Name
T1	$B \bullet 1 = B$	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	$B + B = B$	Idempotency
T4	$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	$B + \overline{B} = 1$	Complements
T6	$B \bullet C = C \bullet B$	$B+C = C+B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	$(B + C) + D = B + (C + D)$	Associativity
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	$B + (C \bullet D) = (B+C)\ (B+D)$ *	Distributivity
T9	$B \bullet (B+C) = B$	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	$(B+C) \bullet (B+\overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) = (B \bullet C) + (\overline{B} \bullet D)$	$(B+C) \bullet (\overline{B}+D) \bullet (C+D) = (B+C) \bullet (\overline{B}+D)$	Consensus
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \ldots} = \overline{B}_0 + \overline{B}_1 + \overline{B}_2 \ldots$	$\overline{B_0 + B_1 + B_2 \ldots} = \overline{B}_0 \bullet \overline{B}_1 \bullet \overline{B}_2 \ldots$	DeMorgan's

* Warning: T8 dual differs from traditional algebra: OR (+) distributes over AND (•)

src: modified from Harris & Harris

Boolean Logic – Formal Laws

- These laws can be used to
  - Reduce the complexity of Boolean equations
  - Convert Boolean equations to a desired form

Convert the following equation to the minimum number of operations

$ab + bc(a + c)$

$b(a + c(a + c))$ ← reverse distributive

$b(a + c)$ ← covering

result requires 2 operations
1 OR and 1 AND

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Convert the following equation to SOP form

$ab + bc(a + c)$ — distributive

$ab + bca + bcc$ — idempotent

$ab + bca + bc$ — reverse distributive

$ab + b(ca + c)$ — consensus

$ab + b(c)$

$ab + bc$