

Digital Logic Reduction

K-maps – SOP form

Last updated 1/7/25

Digital Logic Reduction – K-maps - SOP

- K-Map reduction process – SOP form
 1. Create and populate an appropriately sized K-map
 - 2x2, 2x4, 4x4
 2. 1's Collection
 - Use as few ovals as possible to cover all the 1's
 - Each oval must only contain 1's
 - Ovals may not be diagonal
 - Each oval should be as large as possible
 - Ovals may wrap around the edges of the map
 - Any 1 in the K-map may be included in multiple circles
 3. Interpret the ovals
 - Individual 1's are interpreted as an AND of all inputs
 - Pairs of 1's will include an x and x-bar term, allowing the input to be removed
 - Quads of 1's will include x, x-bar, y, y-bar, terms, allowing those inputs to be removed
 4. Combine the evaluated ovals as an OR term in the reduced logic expression – SOP form

Digital Logic Reduction – K-maps - SOP

- Example 1
 - Create the K map

a	b	c	OUT
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

		AB			
		00	01	11	10
C	0	1	1	1	0
	1	1	0	1	0

Digital Logic Reduction – K-maps - SOP

- Example 1
 - Circle groups
 - There may be multiple collection possibilities

a	b	c	OUT
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

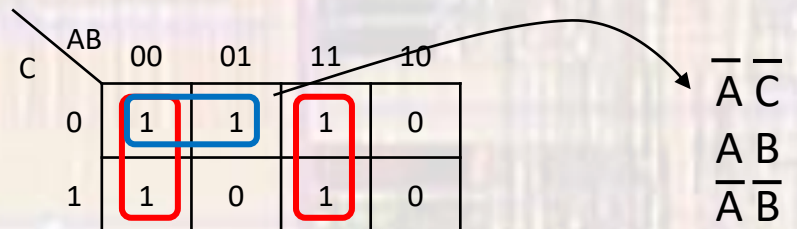
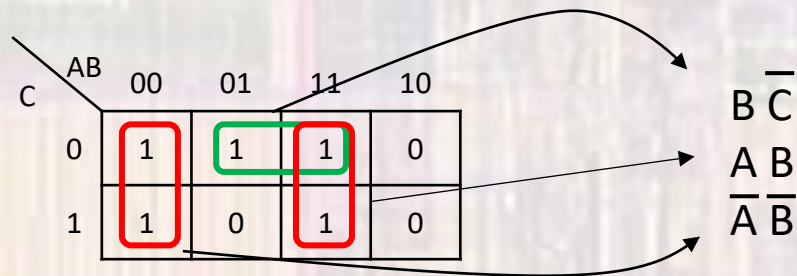
		AB			
		00	01	11	10
C	0	1	1	1	0
	1	1	0	1	0

		AB			
		00	01	11	10
C	0	1	1	1	0
	1	1	0	1	0

Digital Logic Reduction – K-maps - SOP

- Example 1
 - Interpret

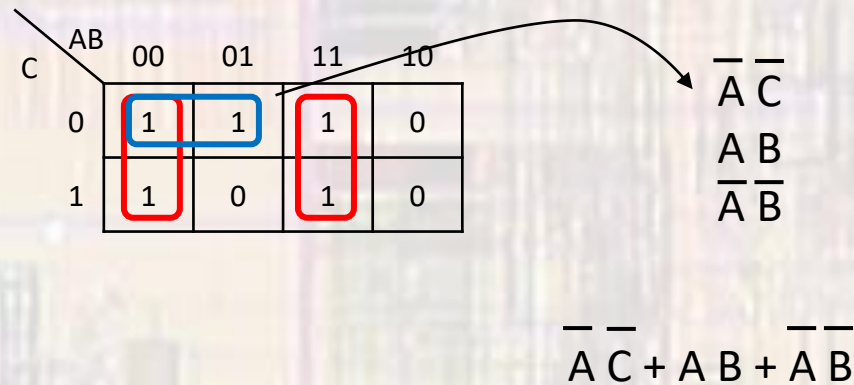
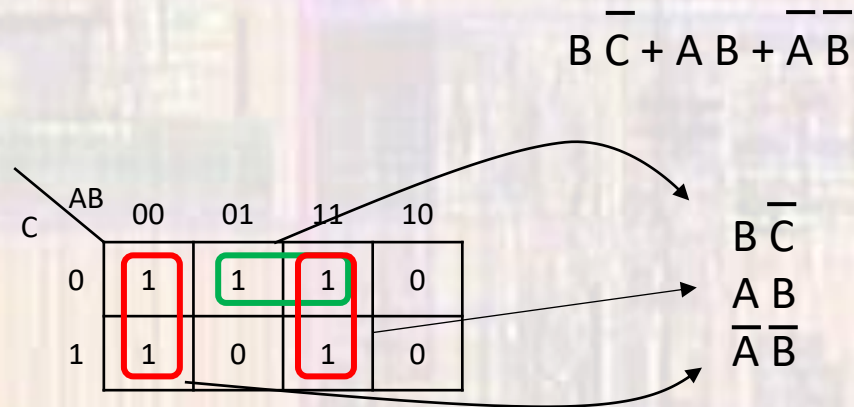
a	b	c	OUT
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1



Digital Logic Reduction – K-maps - SOP

- Example 1
 - Form SOP

a	b	c	OUT
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



Digital Logic Reduction – K-maps - SOP

- Example 2 – create the K-map

a	b	c	d	OUT
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

		AB			
		00	01	11	10
CD	00	1	1	1	0
	01	1	0	1	0
	11	0	0	1	1
	10	1	0	1	1

Digital Logic Reduction – K-maps - SOP

- Example 2 – circle groups
- There may be multiple collection possibilities

a	b	c	d	OUT
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

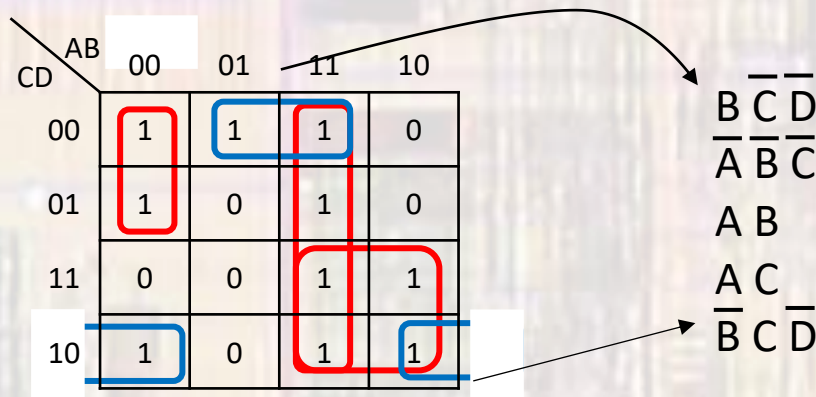
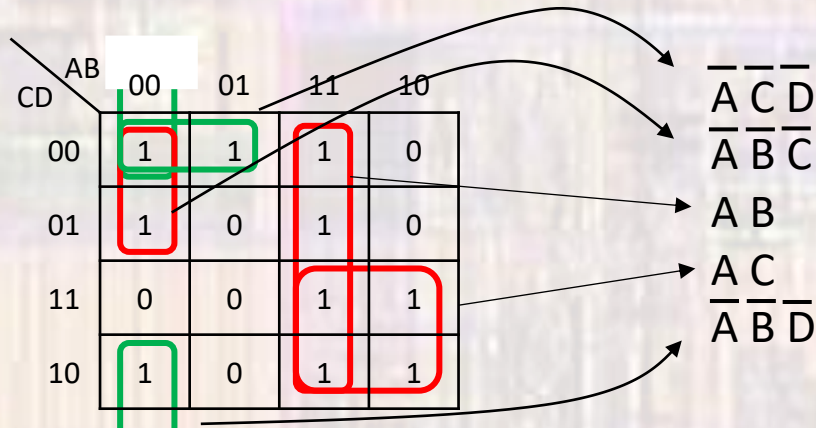
AB \ CD	00	01	11	10
00	1	1	1	0
01	1	0	1	0
11	0	0	1	1
10	1	0	1	1

AB \ CD	00	01	11	10
00	1	1	1	0
01	1	0	1	0
11	0	0	1	1
10	1	0	1	1

Digital Logic Reduction – K-maps - SOP

- Example 2 - interpret

a	b	c	d	OUT
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

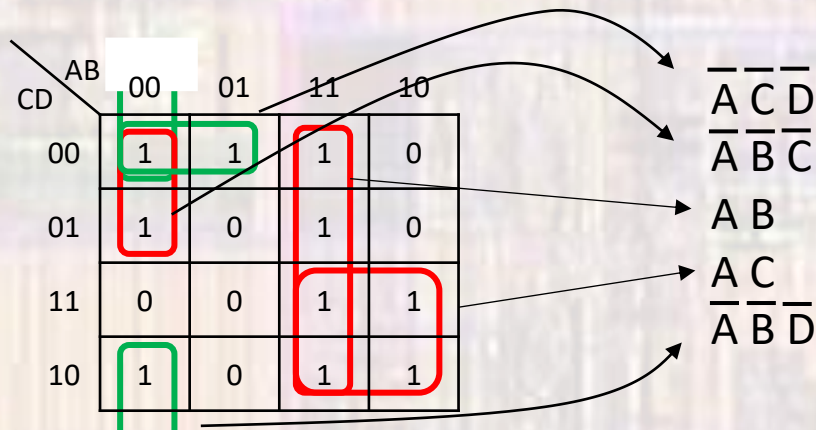


Digital Logic Reduction – K-maps - SOP

- Example 2 – Form SOP

$$\overline{\overline{A}}\overline{\overline{C}}\overline{\overline{D}} + \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}} + AB + AC + \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{D}}$$

a	b	c	d	OUT
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



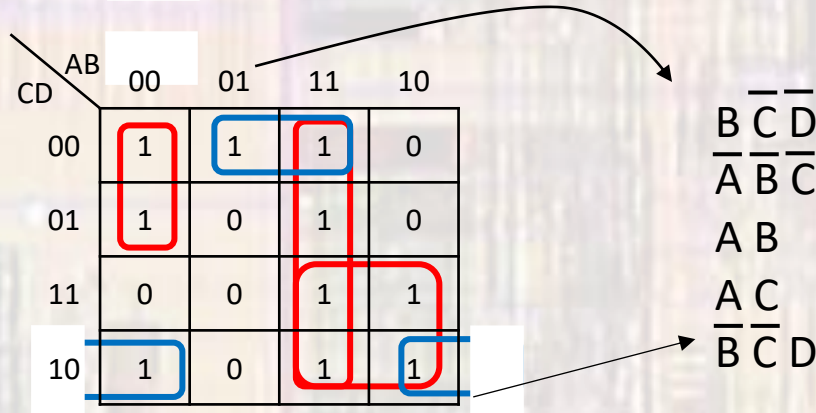
$$\overline{\overline{A}}\overline{\overline{C}}\overline{\overline{D}}$$

$$\overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}}$$

$$AB$$

$$AC$$

$$\overline{\overline{A}}\overline{\overline{B}}\overline{\overline{D}}$$



$$\overline{\overline{B}}\overline{\overline{C}}\overline{\overline{D}}$$

$$\overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}}$$

$$AB$$

$$AC$$

$$\overline{\overline{B}}\overline{\overline{C}}\overline{\overline{D}}$$

$$\overline{\overline{B}}\overline{\overline{C}}\overline{\overline{D}} + \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}} + AB + AC + \overline{\overline{B}}\overline{\overline{C}}\overline{\overline{D}}$$

Digital Logic Reduction – K-maps - SOP

- Example 3 – don't cares – create the K-map

a	b	c	d	OUT
0	0	0	0	1
0	0	0	1	1
0	0	1	0	X
0	0	1	1	X
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	X
1	0	0	0	0
1	0	0	1	0
1	0	1	0	X
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

		AB			
		00	01	11	10
CD	CD	1	1	1	0
	01	1	0	1	0
	11	X	X	1	1
	10	X	0	1	X

Digital Logic Reduction – K-maps - SOP

- Example 3 – don't cares - circle groups
 - There may be multiple collection possibilities

a	b	c	d	OUT
0	0	0	0	1
0	0	0	1	1
0	0	1	0	X
0	0	1	1	X
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	X
1	0	0	0	0
1	0	0	1	0
1	0	1	0	X
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

x's can be treated as 0's or 1's

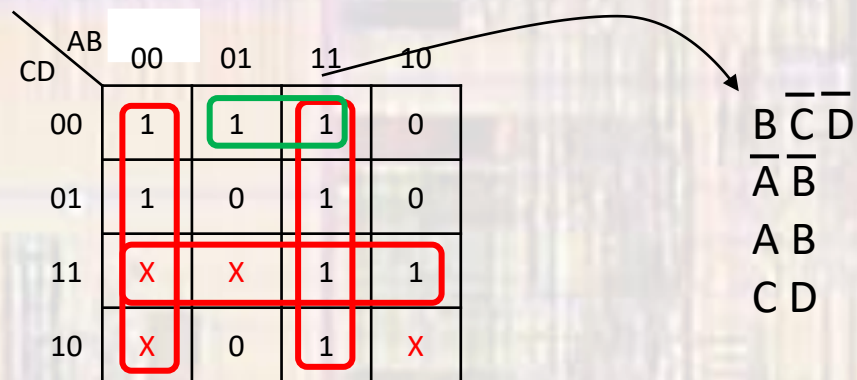
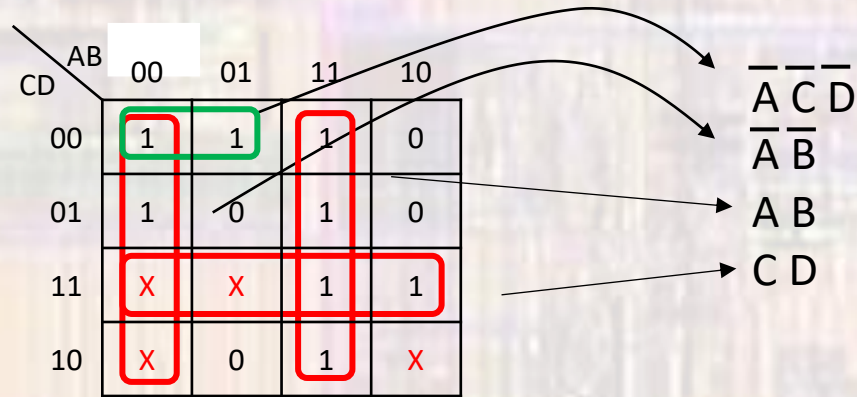
CD \ AB	00	01	11	10
00	1	1	1	0
01	1	0	1	0
11	X	X	1	1
10	X	0	1	X

CD \ AB	00	01	11	10
00	1	1	1	0
01	1	0	1	0
11	X	X	1	1
10	X	0	1	X

Digital Logic Reduction – K-maps - SOP

- Example 3 – don't cares - interpret

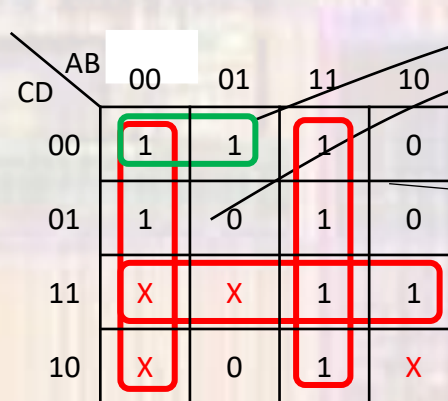
a	b	c	d	OUT
0	0	0	0	1
0	0	0	1	1
0	0	1	0	X
0	0	1	1	X
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	X
1	0	0	0	0
1	0	0	1	0
1	0	1	0	X
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



Digital Logic Reduction – K-maps - SOP

- Example 3 – don't cares – form SOP

a	b	c	d	OUT
0	0	0	0	1
0	0	0	1	1
0	0	1	0	X
0	0	1	1	X
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	X
1	0	0	0	0
1	0	0	1	0
1	0	1	0	X
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



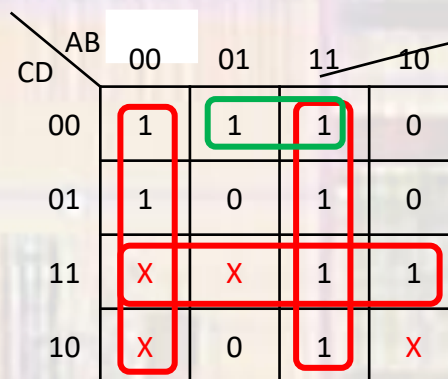
$$\bar{A}\bar{C}\bar{D} + \bar{A}\bar{B} + AB + CD$$

$$\bar{A}\bar{C}\bar{D}$$

$$\bar{A}\bar{B}$$

$$AB$$

$$CD$$



$$B\bar{C}\bar{D}$$

$$\bar{A}\bar{B}$$

$$AB$$

$$CD$$

$$B\bar{C}\bar{D} + \bar{A}\bar{B} + AB + CD$$