Digital Logic Reduction - Laws

Last updated 10/10/24

These slides show reduction of digital logic equations using the formal Boolean laws

Digital Logic Reduction – Formal Laws

- Logic expression laws / identities

Reminder – this is not algebra
+ → OR
* → AND

#	Theorem	Dual	Name
T1	$B \bullet 1 = B$	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	$B + B = B$	Idempotency
T4	$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	$B + \overline{B} = 1$	Complements
T6	$B \bullet C = C \bullet B$	$B+C = C+B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	$(B + C) + D = B + (C + D)$	Associativity
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	$B + (C \bullet D) = (B+C)\ (B+D)$ *	Distributivity
T9	$B \bullet (B+C) = B$	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	$(B+C) \bullet (B+\overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) = (B \bullet C) + (\overline{B} \bullet D)$	$(B+C) \bullet (\overline{B}+D) \bullet (C+D) = (B+C) \bullet (\overline{B}+D)$	Consensus
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \ldots} = \overline{B}_0 + \overline{B}_1 + \overline{B}_2 \ldots$	$\overline{B_0 + B_1 + B_2 \ldots} = \overline{B}_0 \bullet \overline{B}_1 \bullet \overline{B}_2 \ldots$	DeMorgan's

* Warning: T8 dual differs from traditional algebra: OR (+) distributes over AND (•)

src: modified from Harris & Harris

Digital Logic Reduction – Formal Laws

- Example 1
  - Reduce to SOP form

ab + bc(a + c)	
ab + bca + bcc	distributive
ab + bca + bc	idempotent
ab + b(ca + c)	reverse distributive
ab + b(c)	covering
ab + bc	

ab + bc(a + c)	
ab + b(ca + cc)	distributive
ab + b(c)	idempotent
ab + bc	covering
ab + bc	

Digital Logic Reduction – Formal Laws

- Example 2
  - Reduce to SOP form

ab(c+d)+!(a+d)!c	
abc + abd +!(a+d)!c	distributive
abc + abd + (!a!d)!c	DeMorgan's
abc + abd + !a!c!d	commutative

ab(c+d)+$\overline{(a+d)}\,\overline{c}$	
abc + abd +$\overline{(a+d)}\,\overline{c}$	distributive
abc + abd + $(\overline{a}\,\overline{d})\,\overline{c}$	DeMorgan's
abc + abd + $\overline{a}\,\overline{c}\,\overline{d}$	commutative

Digital Logic Reduction – Formal Laws

- Example 3
  - Redo example 2 but with the results in POS form

$$\frac{ab(c+d)+\overline{(a+d)}\,\overline{c}}{\overline{ab(c+d)} * \overline{\overline{(a+d)}\,\overline{c}}}$$

DeMorgan's

$$\frac{}{(\overline{a}+\overline{b}+(\overline{c+d})) * ((a+d)+c)}$$

DeMorgan's

$$\frac{}{(\overline{a}+\overline{b}+\overline{c}\,\overline{d}) * (a+d+c)}$$

DeMorgans

$$\frac{}{\overline{a}a+\overline{a}d+\overline{a}c+\overline{b}a+\overline{b}d+\overline{b}c+\overline{c}\,\overline{d}\,a+\overline{c}\,\overline{d}\,d+\overline{c}\,\overline{d}\,c}$$

distributive

$$\frac{}{0+\overline{a}d+\overline{a}c+\overline{b}a+\overline{b}d+\overline{b}c+\overline{c}\,\overline{d}\,a+0+0}$$

complement

$$(a+\overline{d})(a+\overline{c})(b+\overline{a})(b+\overline{d})(b+\overline{c})(c+d+\overline{a})$$

DeMorgan's

$$(a+\overline{d})(a+\overline{c})(\overline{a}+b)(\overline{a}+c+d)$$

consensus