

Karnaugh-Maps

Last updated 1/7/25

Karnaugh Maps

- Boolean Law Review

- A review of the Boolean Laws reminds us that:

$$A B + A \bar{B} \rightarrow A$$

$$A B C + A B \bar{C} + A \bar{B} C + A \bar{B} \bar{C} \rightarrow A$$

- A closer look at these equations indicates that:
 1. If we can group terms together where only 1 input changes and
 2. We can group: pairs for 2 inputs, quads for 3 inputs, ...

→ We can reduce the equation to a single variable

Karnaugh Maps

- Karnaugh Maps
 - K-maps
 - A systematic tool to reduce logic expressions
- K-Maps use the Boolean logic laws to:
 - Reduce pairs of 2 input variables to a single variable
 - Reduce quads of 3 input variables to a single variable
 - ...
- This is done by encoding the inputs in a fashion that ensures only one input changes between successive cells
 - Gray Code

00 01
10 11

000 010 110 100
001 011 111 101

0000 0100 1100 1000
0001 0101 1101 1001
0011 0111 1111 1011
0010 0110 1110 1010

Karnaugh Maps

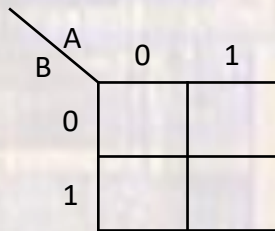
- Creation Process

Given a truth table for the logic expression

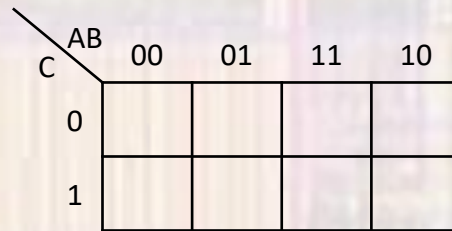
1. Create a $n \times m$ matrix where
 - a) $n + m = \text{number of inputs}$
 - b) $m = n$ or $m = n + 1$
 - c) Label the rows with the n row inputs – see examples
 - d) Label the columns with the m column inputs – see examples
2. Fill in the matrix with the appropriate truth table values

Karnaugh Maps

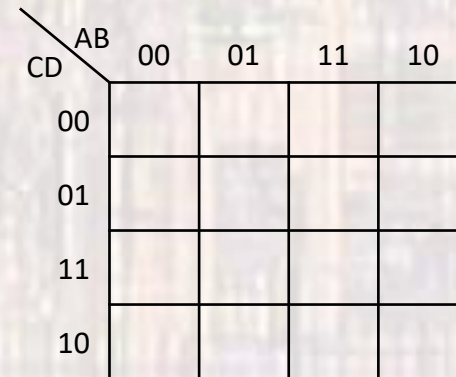
- Karnaugh Maps
 - Map Examples
 - Gray Code



00 01
10 11



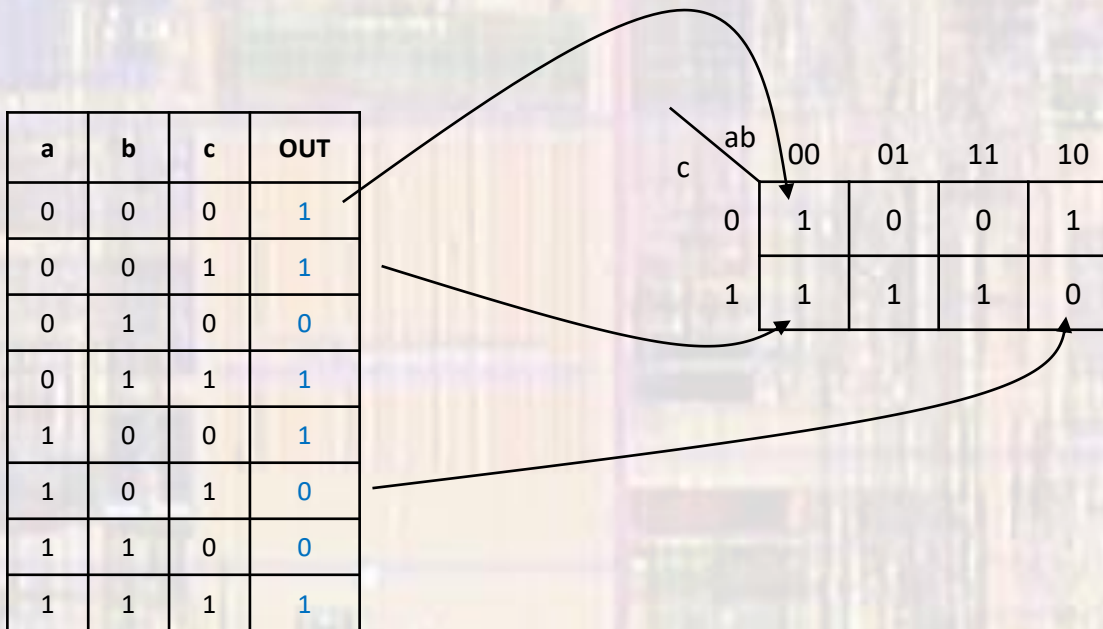
000 010 110 100
001 011 111 101



0000 0100 1100 1000
0001 0101 1101 1001
0011 0111 1111 1011
0010 0110 1110 1010

Karnaugh Maps

- Example 1
 - Creation



Karnaugh Maps

- Example 1
 - Creation – quick fill

a	b	c	OUT
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

		ab			
		00	01	11	10
c	0	1	0	0	1
	1	1	1	1	0

Karnaugh Maps

- Example 2 - create

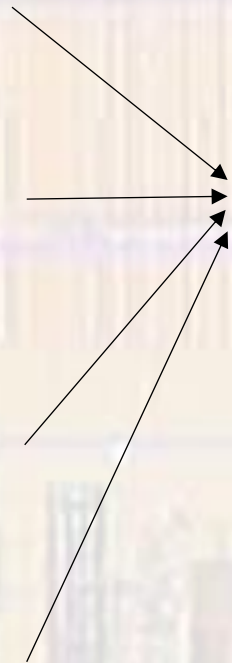
a	b	c	d	OUT
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

cd \ ab	00	01	11	10
00	1	1	1	0
01	1	0	0	0
11	1	1	1	1
10	0	0	1	1

Karnaugh Maps

- Example 2 – create – quick fill

a	b	c	d	OUT
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1



FLIP

cd \ ab	00	01	11	10
00	1	1	1	0
01	1	0	0	0
11	1	1	1	1
10	0	0	1	1

