Signed Binary Numbers

Last updated 10/3/24

These slides introduce signed binary numbers

Signed Binary Numbers

- 3 variations of signed binary numbers
  - Sign-Magnitude
  - One's Complement
  - Two's Complement

- Two's complement is used in almost all digital systems
- We will use the names Two's Complement and Signed interchangeably

Signed Binary Numbers

- Signed Binary (2's complement)
  - Binary representation for a number that can be positive or negative
    - Most data
  - Often just called "signed"
  - Characterized by n-bits
    - I have a 32 bit signed binary number

Signed Binary Numbers

- Bit Values
  - The most significant bit is NOT used to represent the magnitude of the value
  - The most significant bit INDICATES the sign but is NOT a sign bit

- MSB = 0 → indicates a positive value
- MSB = 1 → indicates a negative value

Signed Binary Numbers

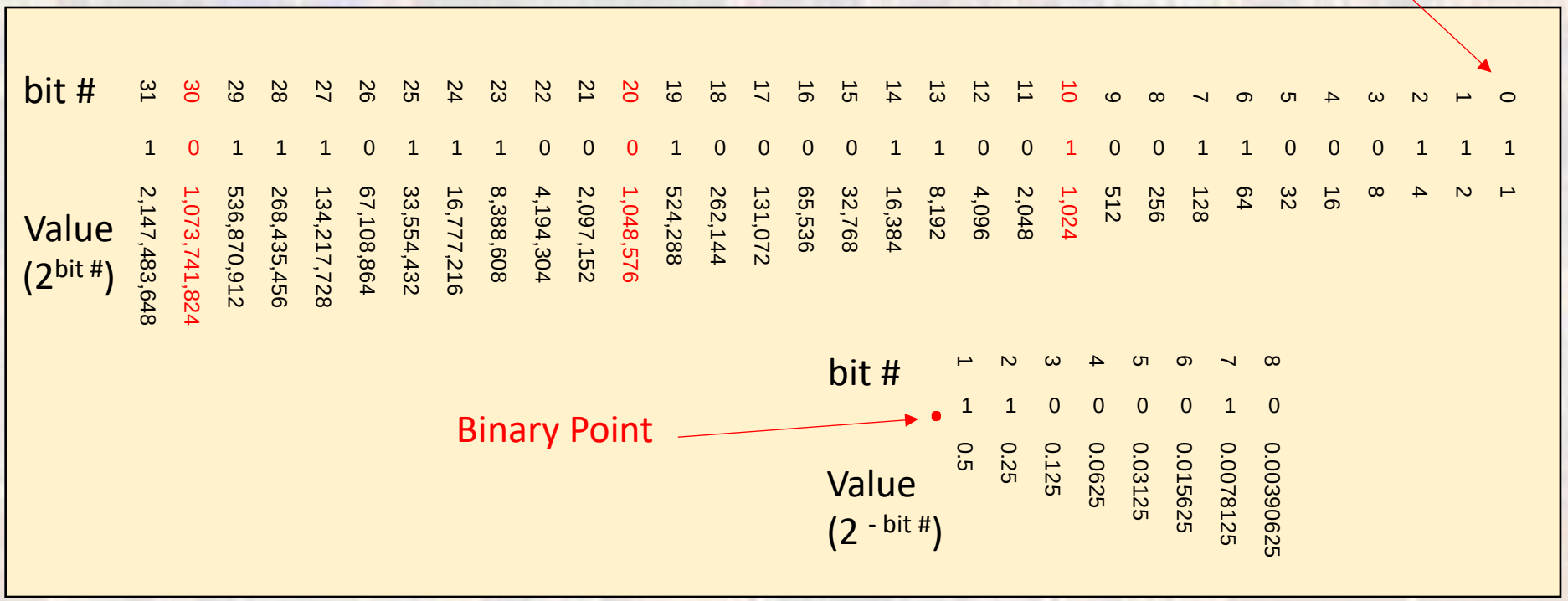
- Number formation
  - Positive numbers are formed in normal binary format
    - Excluding the msb – it is not used to create the binary value
    - The msb will always be 0
  - Negative numbers are formed by
    1) Creating the positive binary number
    2) Flipping all bits and adding 1

Signed Binary Numbers

- Binary Bit Values
  - Excluding the MSB

Note: we start counting bits at 0

Right to Left



Left to Right

Signed Binary Numbers

- Convert Signed Binary to Decimal
  - Process:
    - If the MSB is 0 :
      - Convert the value to decimal using the positional conversion process used for unsigned binary conversion
    - If the MSB is 1 :
      - Write down the minus sign (so you don't forget)
      - Flip all the individual bits and add 1
      - Convert the value to decimal using the positional conversion process used for unsigned binary conversion

Signed Binary Numbers

- Convert Signed Binary to Decimal

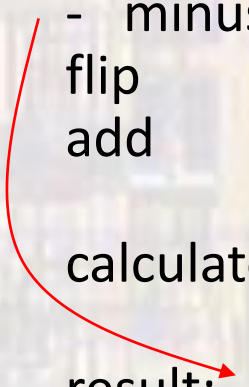
Convert signed binary 0011 0010 to decimal

MSB is 0 → positive → just calculate

0011 0010 → 32 + 16 + 2 = 50

Convert signed binary 1100 1110 to decimal

MSB is 1 → negative → write -, flip and add 1, calculate value



- minus sign

flip 0011 0001

add +0000 0001

0011 0010

calculate value → 32 + 16 + 2 = 50

result: -50

Signed Binary Numbers

- Convert Decimal to Signed Binary
  - Process:
    - If the decimal value is positive
      - Convert the value to binary using the positional conversion process used for unsigned binary conversion, but exclude the MSB value, set it to 0
    - If the decimal value is negative
      - Convert the value to decimal using the positional conversion process used for unsigned binary conversion
      - Flip all the individual bits and add 1

This is the opposite of converting signed binary to decimal

Signed Binary Numbers

- Convert Decimal to Signed Binary

Convert 45 to 8-bit signed binary

Decimal is positive → just calculate but exclude the msb → 0

45		45	45	13	13	5	1	1
	0(128)	0(64)	1(32)	0(16)	1(8)	1(4)	0(2)	1(1)
45		45	13	13	5	1	1	0

Result: 0010 1101

Convert -45 to 8-bit signed binary

Decimal is negative → calculate value, flip and add 1

calculate value: 0010 1101 from above

flip 1101 0010

add +0000 0001

1101 0011

result: 1101 0011

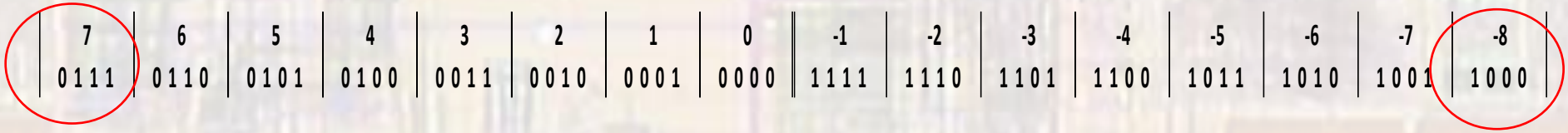
Signed Binary Numbers

- Word Length Extension - Sign Extension
  - When extending to larger word sizes, extend the MSB to the left

4 bit		8 bit		16 bit
0110	→	00000110	→	0000000000000110
1001	→	11111001	→	1111111111111001

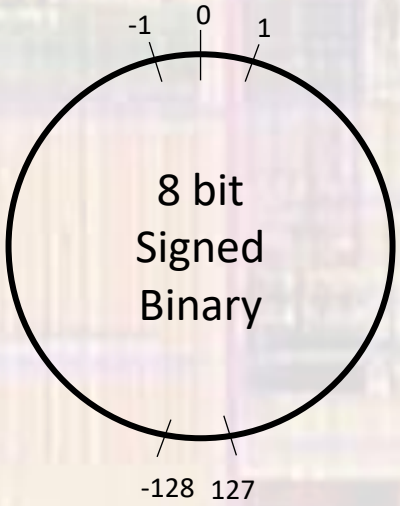
Signed Binary Numbers

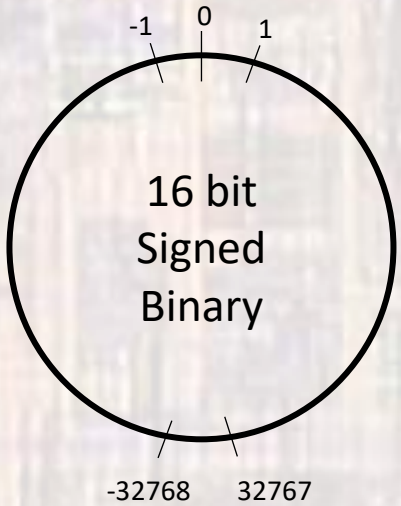
- Limits
  - Maximum values:
    - 4 bits = +7, -8 = $2^3-1, -2^3$
    - 8 bits = + 127, -128 = $2^7-1, -2^7$
    - 16 bits = + 32,767, -32,768 = $2^{15}-1, -2^{15}$
    - Not Symmetric



Signed Binary Numbers

- Number Lines
  - Fixed size limitations on binary numbers causes the number line to wrap around





0111	7
+ 0001	1
1000	-8

01111111	127
+ 00000001	1
10000000	-128

0111111111111111	32767
+ 0000000000000001	1
1000000000000000	-32768

Signed Binary Numbers

- 2's complement notation
  - Advantages
    - Addition is done the same way as unsigned numbers – same adder circuit
    - ONLY 1 ZERO !
    - Simple word length extension
  - Disadvantages
    - Asymmetric range
    - Harder to do comparisons
    - Not intuitive