

State Machine Encoding

Last updated 12/12/24

State Machine Encoding

- Encoding

Definition of ENCODE (WEBSTER)

transitive verb

1a : to convert (as a body of information) from one system of communication into another; *especially* : to convert (a message) into code

- Morse Code
- Braille

State Machine Encoding

- Binary (standard)
- Represent a set of values (states, light colors, directions ...) with a standard binary sequence
 - $S_0 \rightarrow 00$, $S_1 \rightarrow 01$, $S_2 \rightarrow 10$, $S_3 \rightarrow 11$
 - Red $\rightarrow 00$, Yellow $\rightarrow 01$, Green $\rightarrow 10$
 - Forward $\rightarrow 00$, Turn right $\rightarrow 01$, Turn left $\rightarrow 10$
- m values require $\text{ceil}(\log_2 m)$ code bits
- n code bits can represent 2^n values

State Machine Encoding

- Binary
- Stop light example

Current State S	Inputs		Next State S'
	T _{NS}	T _{EW}	
ST0	0	x	ST1
ST0	1	x	ST0
ST1	x	x	ST2
ST2	x	0	ST3
ST2	x	1	ST2
ST3	x	x	ST0

Current State S	Current State		Inputs		Next State	
	S1	S0	T _{NS}	T _{EW}	S1'	S0'
ST0	0	0	0	x	0	1
ST0	0	0	1	x	0	0
ST1	0	1	x	x	1	0
ST2	1	0	x	0	1	1
ST2	1	0	x	1	1	0
ST3	1	1	x	x	0	0



State Machine Encoding

- One Hot
 - Represent a set of values (states, light colors, directions ...) with a binary word with only one bit high for each code
 - $S_0 \rightarrow 0001$, $S_1 \rightarrow 0010$, $S_2 \rightarrow 0100$, $S_3 \rightarrow 1000$
 - Red $\rightarrow 001$, Yellow $\rightarrow 010$, Green $\rightarrow 100$
 - Forward $\rightarrow 001$, Turn right $\rightarrow 010$, Turn left $\rightarrow 100$
 - m values require m code bits
 - n code bits can represent n values

State Machine Encoding

- One-hot
- Stop light example

Current State S	Inputs		Next State S'
	T _{NS}	T _{EW}	
ST0	0	x	ST1
ST0	1	x	ST0
ST1	x	x	ST2
ST2	x	0	ST3
ST2	x	1	ST2
ST3	x	x	ST0

Current State S	Current State				Inputs		Next State			
	S3	S2	S1	S0	T _{NS}	T _{EW}	S3'	S2'	S1'	S0'
ST0	0	0	0	1	0	x	0	0	1	0
ST0	0	0	0	1	1	x	0	0	0	1
ST1	0	0	1	0	x	x	0	1	0	0
ST2	0	1	0	0	x	0	1	0	0	0
ST2	0	1	0	0	x	1	0	1	0	0
ST3	1	0	0	0	x	x	0	0	0	1



State Machine Encoding

- One Not (one cold)
 - Represent a set of values (states, light colors, directions ...) with a binary word with only one bit low for each code
 - $S_0 \rightarrow 1110$, $S_1 \rightarrow 1101$, $S_2 \rightarrow 1011$, $S_3 \rightarrow 0111$
 - Red $\rightarrow 110$, Yellow $\rightarrow 101$, Green $\rightarrow 011$
 - Forward $\rightarrow 110$, Turn right $\rightarrow 010$, Turn left $\rightarrow 011$
 - m values require m code bits
 - n code bits can represent n values

State Machine Encoding

- One-hot (one-cold)
- Stop light example

Current State S	Inputs		Next State S'
	T _{NS}	T _{EW}	
ST0	0	x	ST1
ST0	1	x	ST0
ST1	x	x	ST2
ST2	x	0	ST3
ST2	x	1	ST2
ST3	x	x	ST0

Current State S	Current State				Inputs		Next State			
	S3	S2	S1	S0	T _{NS}	T _{EW}	S3'	S2'	S1'	S0'
ST0	1	1	1	0	0	x	1	1	0	1
ST0	1	1	1	0	1	x	1	1	1	0
ST1	1	1	0	1	x	x	1	0	1	1
ST2	1	0	1	1	x	0	0	1	1	1
ST2	1	0	1	1	x	1	1	0	1	1
ST3	0	1	1	1	x	x	1	1	1	0



State Machine Encoding

- Gray
 - Represent a set of values (states, light colors, directions ...) with a binary word with only one bit different between adjoining codes - twisted shift register
 - $S_0 \rightarrow 00$, $S_1 \rightarrow 10$, $S_2 \rightarrow 11$, $S_3 \rightarrow 01$
 - $A \rightarrow 000$, $B \rightarrow 001$, $C \rightarrow 011$, $D \rightarrow 010$,
 $E \rightarrow 110$, $F \rightarrow 111$, $G \rightarrow 101$, $H \rightarrow 100$
 - m values require $\text{ceil}(\log_2 m)$ code bits
 - n code bits can represent 2^n values

State Machine Encoding

- Gray
- Stop light example

Current State S	Inputs		Next State S'
	T _{NS}	T _{EW}	
ST0	0	x	ST1
ST0	1	x	ST0
ST1	x	x	ST2
ST2	x	0	ST3
ST2	x	1	ST2
ST3	x	x	ST0

Current State S	Current State		Inputs		Next State	
	S1	S0	T _{NS}	T _{EW}	S1'	S0'
ST0	0	0	0	x	0	1
ST0	0	0	1	x	0	0
ST1	0	1	x	x	1	1
ST2	1	1	x	0	1	0
ST2	1	1	x	1	1	1
ST3	1	0	x	x	0	0



State Machine Encoding

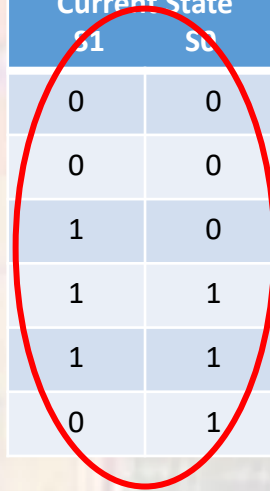
- Johnson
 - Represent a set of values (states, light colors, directions ...) with a binary word with only one bit different between adjoining codes - twisted shift register
 - $S_0 \rightarrow 00$, $S_1 \rightarrow 10$, $S_2 \rightarrow 11$, $S_3 \rightarrow 01$
 - $A \rightarrow 000$, $B \rightarrow 100$, $C \rightarrow 110$, $D \rightarrow 111$,
 $E \rightarrow 011$, $F \rightarrow 001$
 - m values require $\text{ceil}(m/2)$ code bits
 - n code bits can represent 2^n values

State Machine Encoding

- Johnson
- Stop light example

Current State S	Inputs		Next State S'
	T _{NS}	T _{EW}	
ST0	0	x	ST1
ST0	1	x	ST0
ST1	x	x	ST2
ST2	x	0	ST3
ST2	x	1	ST2
ST3	x	x	ST0

Current State S	Current State		Inputs		Next State	
	S1	S0	T _{NS}	T _{EW}	S1'	S0'
ST0	0	0	0	x	1	0
ST0	0	0	1	x	0	0
ST1	1	0	x	x	1	1
ST2	1	1	x	0	0	1
ST2	1	1	x	1	1	1
ST3	0	1	x	x	0	0



State Machine Encoding

- Comparison – considerations
 - Memory elements (bits) \rightarrow area or fit
 - Coding / Decoding logic \rightarrow area or fit
 - Speed
 - Shorter logic chains \rightarrow faster maximum clock speeds
 - Power
 - $P = v * i$
 - $P = v * c * dv/dt$
 - dv/dt represents a transition $0 \rightarrow 1$
 - Fewer transitions \rightarrow lower power