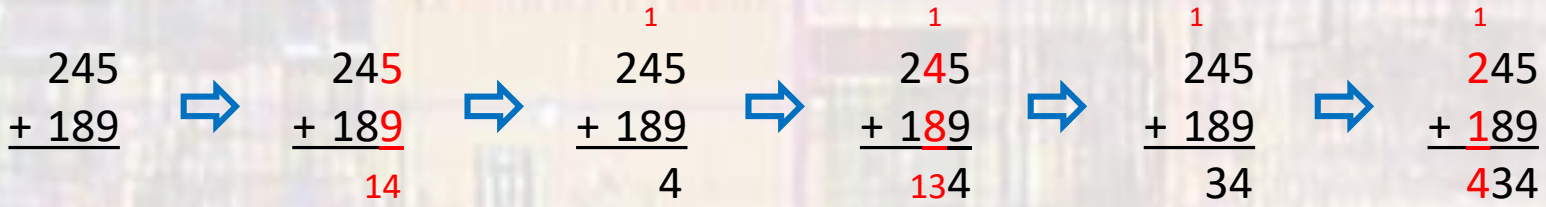
# Binary Addition & Subtraction

Common – last updated 11/7/18

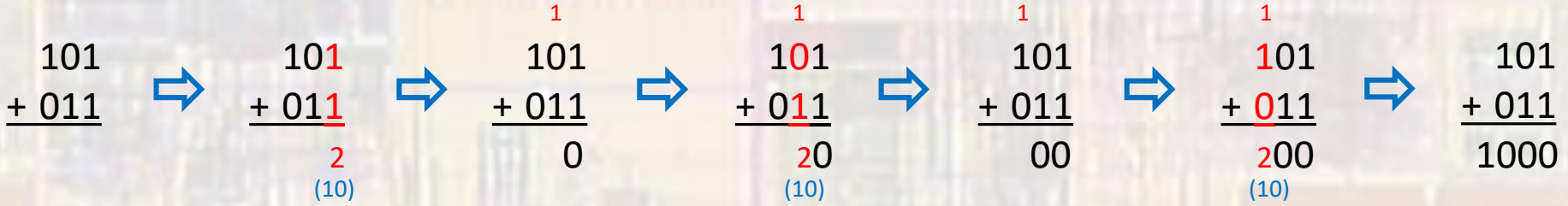
# Addition

- Elementary school concepts
  - Add columns of numbers and keep track of the carry over to the next column
  - We normally use the decimal number system
    - Digits: 0-9
    - Carry over is in sets of 10x



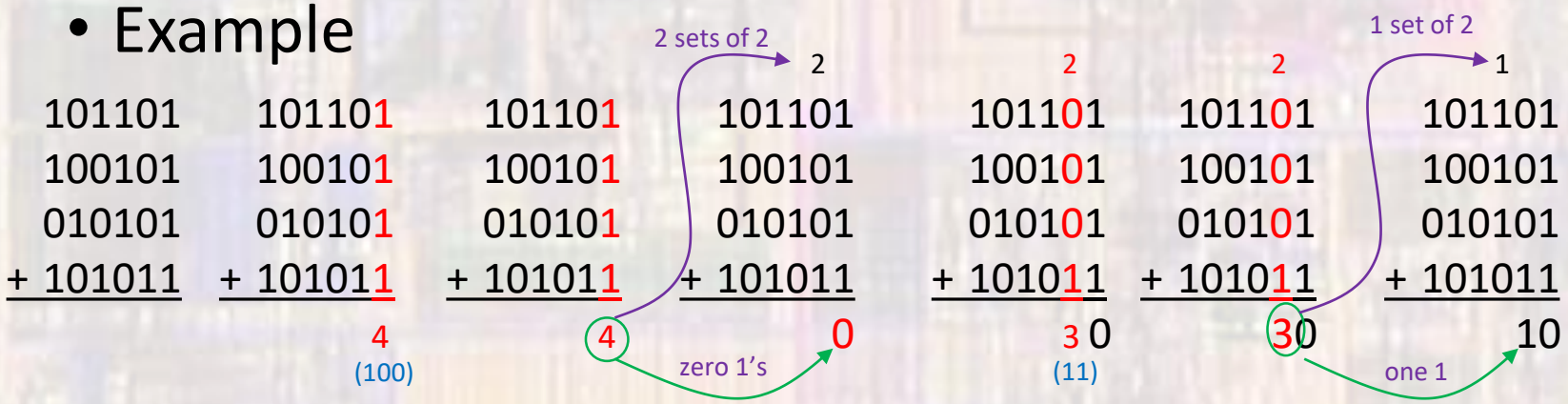
# Binary Addition - unsigned

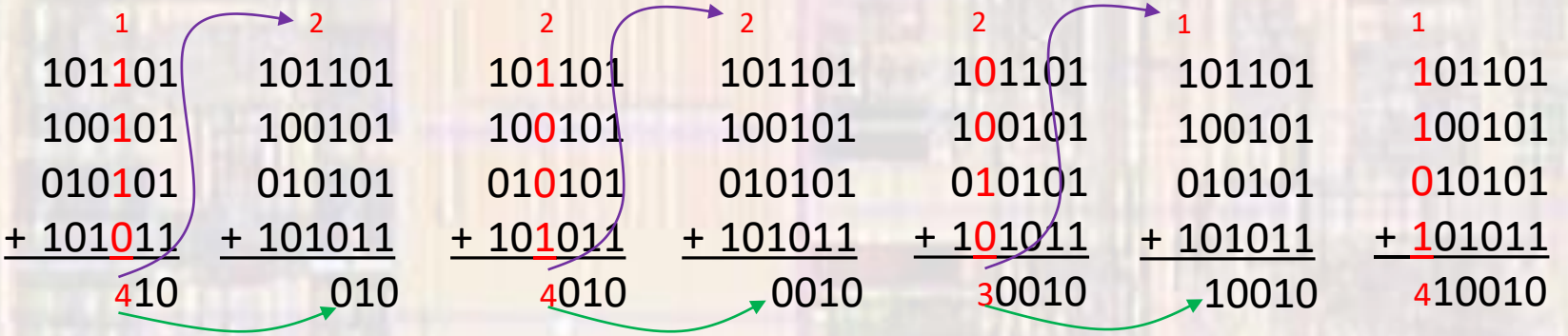
- Extend elementary school concepts
  - Add columns of numbers and keep track of the carry over to the next column
  - Use the binary number system
    - Digits: 0-1
    - Carry over is in sets of 2x

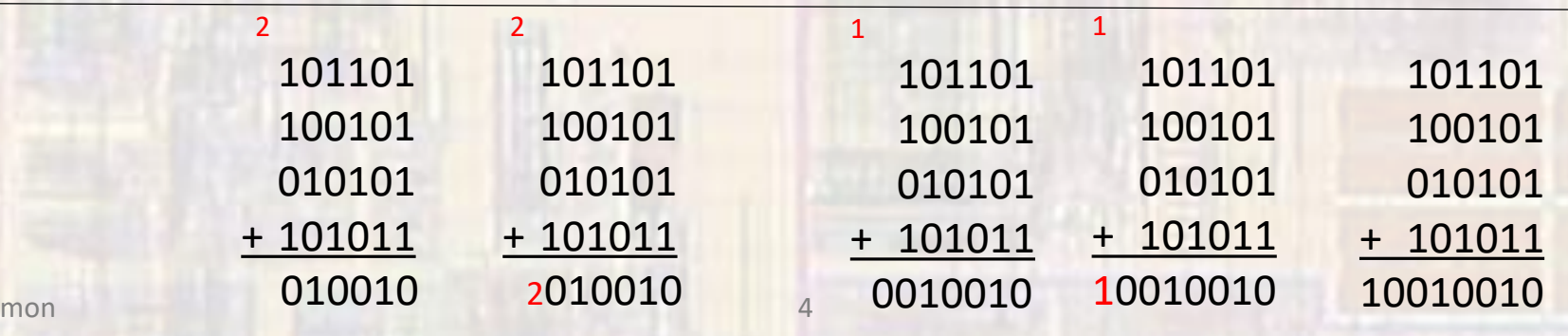


# Binary Addition - unsigned

- Example



# Binary Addition - unsigned

- Overflow
  - In elementary school we did not care how many digits the answer required

```
  745
+ 589
 1334
```

  - In binary addition – we are generally representing something that ultimately is to be executed in hardware
    - Our hardware cannot change the number of bits (wires) it can hold
    - We must establish a maximum number size and create an error when the result of the addition exceeds this size
    - The error is called an overflow

3 bit addition

```
  101
+ 011
 1000
```

overflow

5 bit addition

```
  00101
+ 00011
  01000
```

OK

6 bit addition

```
  101101
  100101
  010101
+ 101011
 10010010
```

overflow

8 bit addition

```
  00101101
  00100101
  00010101
+ 00101011
  10010010
```

OK

# Binary Addition - signed

- Signed binary addition is done the same way as unsigned addition
  - Signed – means 2's complement representation
  - Overflow is determined differently
    - carry-in of the msb ≠ carry-out of the msb → overflow

8 bit signed addition

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | | 1 | 1 | 1 | 1 | 1 | | | |
| | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 71 |
| + | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | + 29 |
| | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 100 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | |
| | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 103 |
| + | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | + 61 |
| | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 164 |

overflow -28

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | | | | 1 | 1 | 1 | | |
| | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | -57 |
| + | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | + -11 |
| | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | -68 |

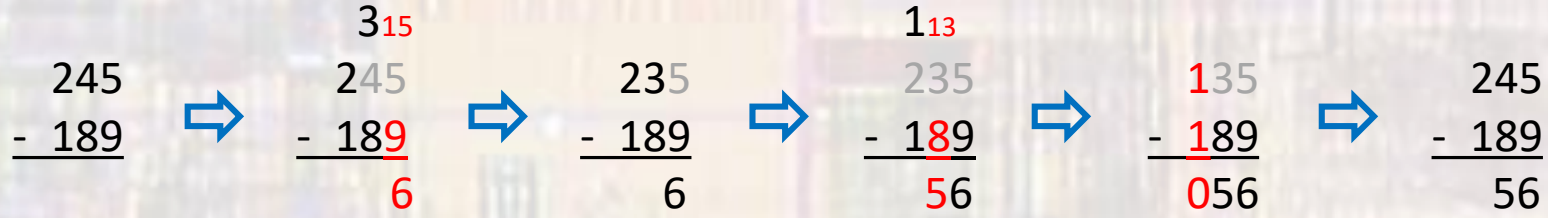
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | 1 | 1 | 1 | | |
| | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | -57 |
| + | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | + -123 |
| | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | -180 |

overflow 68

# Subtraction

- Elementary school concepts
  - Subtract columns of numbers and keep track of how much is borrowed from the next column
  - This is very difficult to implement in hardware

```
  245   minuend
- 189   subtrahend
-----
   56   difference
```



# Binary Subtraction - signed

- Binary Subtraction - signed
  - Signed – means 2's complement representation
  - Negate the subtrahend and add
  - Overflow – same rule during addition

```
   71          71
 - 29    ⇨   + (-29)
```

```
    0 1 0 0 0 1 1 1        71
 -  0 0 0 1 1 1 0 1     -  29
```

⇩

```
 1  1           1 1 1
    0 1 0 0 0 1 1 1        71
 +  1 1 1 0 0 0 1 1     + (-29)
    0 0 1 0 1 0 1 0        42
```

```
   7          7
 - (-99)  ⇨   + 99
```

```
    0 0 0 0 0 1 1 1         7
 -  1 0 0 1 1 1 0 1     - (-99)
```

⇩

```
 0        1 1 1
    0 0 0 0 0 1 1 1         7
 +  0 1 1 0 0 0 1 1     +  99
    0 1 1 0 1 0 1 0       116
```