

# Binary Addition & Subtraction

Common – last updated  
11/7/18

# Addition

- Elementary school concepts
  - Add columns of numbers and keep track of the carry over to the next column
  - We normally use the decimal number system
    - Digits: 0-9
    - Carry over is in sets of 10x

$$\begin{array}{r} 245 \\ + 189 \\ \hline \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 245 \\ + 189 \\ \hline 14 \\ \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 245 \\ + 189 \\ \hline 4 \\ \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 245 \\ + 189 \\ \hline 134 \\ \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 245 \\ + 189 \\ \hline 34 \\ \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 245 \\ + 189 \\ \hline 434 \\ \end{array}$$

# Binary Addition - unsigned

- Extend elementary school concepts
  - Add columns of numbers and keep track of the carry over to the next column
  - Use the binary number system
    - Digits: 0-1
    - Carry over is in sets of 2x

$$\begin{array}{r} 101 \\ + 011 \\ \hline \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 10\textcolor{red}{1} \\ + 01\textcolor{red}{1} \\ \hline \textcolor{red}{2} \\ (10) \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 101 \\ + 011 \\ \hline 0 \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 101 \\ + 011 \\ \hline 20 \\ (10) \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 101 \\ + 011 \\ \hline 00 \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 101 \\ + 011 \\ \hline 200 \\ (10) \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 101 \\ + 011 \\ \hline 1000 \end{array}$$

# Binary Addition - unsigned

- Example

$$\begin{array}{r}
 101101 \quad 101101 \quad 101101 \\
 100101 \quad 100101 \quad 100101 \\
 010101 \quad 010101 \quad 010101 \\
 + 101011 \quad + 101011 \quad + 101011 \\
 \hline
 & 4 \quad (100) & 4 \quad \text{zero 1's} \quad 0 \\
 & & \text{2 sets of 2} \quad 2 \\
 & & 101101 \quad 101101 \\
 & & 100101 \quad 100101 \\
 & & 010101 \quad 010101 \\
 & & + 101011 \quad + 101011 \\
 & & \hline
 & 3 \quad (11) & 3 \quad 0 \\
 & & \text{2} \quad 2 \\
 & & 101101 \quad 101101 \\
 & & 100101 \quad 100101 \\
 & & 010101 \quad 010101 \\
 & & + 101011 \quad + 101011 \\
 & & \hline
 & 3 \quad 0 & 10 \\
 & & \text{1 set of 2} \quad 1 \\
 & & 101101 \\
 & & 100101 \\
 & & 010101 \\
 & & + 101011 \\
 & & \hline
 & & 10
 \end{array}$$

$$\begin{array}{r}
 101101 \quad 101101 \quad 101101 \quad 101101 \quad 101101 \\
 100101 \quad 100101 \quad 100101 \quad 100101 \quad 100101 \\
 010101 \quad 010101 \quad 010101 \quad 010101 \quad 010101 \\
 + 101011 \quad + 101011 \quad + 101011 \quad + 101011 \quad + 101011 \\
 \hline
 410 \quad 010 \quad 4010 \quad 0010 \quad 30010 \\
 & \text{2} & \text{2} & 1 & 1 \\
 & & \text{2} \quad 2 \\
 & & 101101 \quad 101101 \\
 & & 100101 \quad 100101 \\
 & & 010101 \quad 010101 \\
 & & + 101011 \quad + 101011 \\
 & & \hline
 & & 10010 \\
 & & \text{4} \quad 10010
 \end{array}$$

$$\begin{array}{r}
 101101 \quad 101101 \quad 101101 \quad 101101 \quad 101101 \\
 100101 \quad 100101 \quad 100101 \quad 100101 \quad 100101 \\
 010101 \quad 010101 \quad 010101 \quad 010101 \quad 010101 \\
 + 101011 \quad + 101011 \quad + 101011 \quad + 101011 \quad + 101011 \\
 \hline
 010010 \quad 2010010 \quad 0010010 \quad 10010010 \quad 10010010
 \end{array}$$

# Binary Addition - unsigned

- Overflow
  - In elementary school we did not care how many digits the answer required
  - In binary addition – we are generally representing something that ultimately is to be executed in hardware
    - Our hardware cannot change the number of bits (wires) it can hold
    - We must establish a maximum number size and create an error when the result of the addition exceeds this size
    - The error is called an **overflow**

$$\begin{array}{r} 745 \\ + 589 \\ \hline 1334 \end{array}$$

3 bit addition

$$\begin{array}{r} 101 \\ + 011 \\ \hline 1000 \end{array}$$

overflow

5 bit addition

$$\begin{array}{r} 00101 \\ + 00011 \\ \hline 01000 \end{array}$$

OK

6 bit addition

$$\begin{array}{r} 101101 \\ + 101011 \\ \hline 10010010 \end{array}$$

overflow

8 bit addition

$$\begin{array}{r} 00101101 \\ + 00101011 \\ \hline 10010010 \end{array}$$

OK

# Binary Addition - signed

- Signed binary addition is done the same way as unsigned addition
  - Signed – means 2's complement representation
  - Overflow is determined differently
    - carry-in of the msb  $\neq$  carry-out of the msb  $\rightarrow$  overflow

8 bit signed addition

$$\begin{array}{r} \boxed{0} & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ + & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ \hline 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{array} \quad \begin{array}{r} 71 \\ + 29 \\ \hline 100 \end{array}$$

$$\begin{array}{r} \boxed{0} & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ + & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ \hline 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{array} \quad \begin{array}{r} 103 \\ + 61 \\ \hline 164 \end{array}$$

overflow

-28

$$\begin{array}{r} \boxed{1} & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ + & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ \hline 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{array} \quad \begin{array}{r} -57 \\ + -11 \\ \hline -68 \end{array}$$

$$\begin{array}{r} \boxed{1} & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ + & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{array} \quad \begin{array}{r} -57 \\ + -123 \\ \hline -180 \end{array}$$

overflow

68

# Subtraction

- Elementary school concepts
  - Subtract columns of numbers and keep track of how much is borrowed from the next column
  - This is very difficult to implement in hardware

$$\begin{array}{r} 245 \text{ minuend} \\ - 189 \text{ subtrahend} \\ \hline 56 \text{ difference} \end{array}$$

$$\begin{array}{r} 245 \\ - 189 \\ \hline 6 \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 245 \\ - 189 \\ \hline 6 \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 235 \\ - 189 \\ \hline 6 \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 235 \\ - 189 \\ \hline 56 \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 135 \\ - 189 \\ \hline 056 \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{r} 245 \\ - 189 \\ \hline 56 \end{array}$$

# Binary Subtraction - signed

- Binary Subtraction - signed
  - Signed – means 2's complement representation
  - Negate the subtrahend and add
  - Overflow – same rule during addition

$$\begin{array}{r} 71 \\ - 29 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{r} 71 \\ + (-29) \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ - (-99) \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{r} 7 \\ + 99 \\ \hline \end{array}$$

$$\begin{array}{r} 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ - 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ \hline \end{array} \quad \begin{array}{r} 71 \\ - 29 \\ \hline \end{array}$$

↓

$$\boxed{1} \quad \begin{array}{r} 1 & & & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ + 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ \hline 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{array} \quad \begin{array}{r} 71 \\ + (-29) \\ \hline 42 \end{array}$$

$$\begin{array}{r} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ - 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ - (-99) \\ \hline \end{array}$$

↓

$$\boxed{0} \quad \begin{array}{r} 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ + 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ \hline 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{array} \quad \begin{array}{r} 7 \\ + 99 \\ \hline 116 \end{array}$$