# Number Systems
# Floating Point

Last updated 8/20/20

# Number Systems

- Scientific Number Representation
  - $1.60217657 \times 10^{-19}$ coulombs
  - $6.0221413 \times 10^{+23}$ units/mole
  - Normalized to have only 1 digit (non-zero) to the left of the decimal point
  - multiplied by a power of 10
  - 5692.3456 → $5.6923456 \times 10^{+3}$
  - .00023456 → $2.3456 \times 10^{-4}$
  - format is: mantissa $\times 10^{\text{exponent}}$

# Number Systems

- Binary Floating Point Number Representation
  - Normalized to have only 1 digit to the left of the decimal point
    - this must be a 1 since our choices are only 0 and 1 and we don't use 0
  - multiplied by a power of 2

  - 1011.1101 → $1.0111101 \times 2^{+3}$
  - .00011001 → $1.1001 \times 2^{-4}$

  - format is: mantissa × $2^{exponent}$

BUT

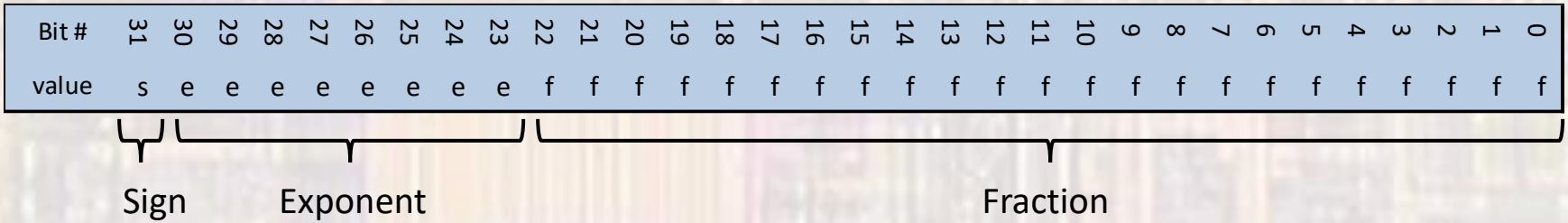
- since the mantissa always starts with "1." we can use
  1.fraction × $2^{exponent}$

# Number Systems

- Binary Floating Point Number Representation
  - It is simpler to work with only positive exponents
  - Bias the exponent
    - With an 8 bit exponent the range is:
      +127 to -127 using signed magnitude notation
    - Add 127 to the desired exponent value (for use in the representation)
      actual range is still +127 to -127
      representation range is 254 to 0
    - called an exponent with +127 bias
  - format is now: value = 1.fraction × $2^{(exponent - 127)}$
    desired value representation

# Number Systems

- Binary Floating Point Number Representation
  - IEEE Standard
    - value = (-1 x sign) x 1.fraction × $2^{(exponent - 127)}$
    - 32 bit format




  - Special cases
    - If E = 255, and F is non-zero, then the value is NaN (Not a Number)
    - If E = 255, F = 0 and S = 1, then the value is -infinity
    - If E = 255, F = 0, and S = 0, then the value is +infinity
    - If E = 0, and F = 0, then the value is 0
  - Range
    - $1.11111111111111111111111_2 \times 2^{+127} = 3.4028 \times 10^{38}$
    - $1.00000000000000000000001_2 \times 2^{-127} = 1.1754 \times 10^{-38}$
    - 24 bit fractional precision ↔ 6 to 7 decimal digits

# Number Systems

- Example

use IEEE standard floating point to represent: 2,345,678.7109375

2,345,678 = 0010 0011 1100 1010 1100 1110 = 0x23CACE
0.7109375 = 0.10110110 = 0x0.B6

2,345,678.7109375 = 0010 0011 1100 1010 1100 1110 . 1011 0110
= 1. 0 0011 1100 1010 1100 1110 1011 0110 × $2^{21}$

fraction = 0001 1110 0101 0110  0111 0101 1011 0
exponent = 21 + 127 = 148 = 1001 0100
sign = 0

will not fit in fraction part of the notation

0  10010100  0001 1110 0101 0110 0111 010

# Number Systems

- Example

convert the IEEE floating point number
0 10010100 0001 1110 0101 0110 0111 010 to decimal

sign = 0
exponent = 1001 0100 = 148 → $2^{148-127} = 2^{21}$
fraction = 0001 1110 0101 0110 0111 010

+ 1.0001 1110 0101 0110 0111 010 × $2^{21}$
= 1 0001 1110 0101 0110 01110 . 10
= 2345678.5

error = (0.5 – 0.7109375)/2345678.5 = - $9x10^{-8}$
~7 decimal digits of precision