Number Systems
Two's Complement

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Number Systems

- Two's Complement
  - Negative numbers are formed by flipping all bits and adding 1
  - Positive numbers are formed in normal binary format
  - Most Significant Bit (MSB) represents the sign
    (but it is NOT a sign bit)
    - MSB = 0 → positive
    - MSB = 1 → negative
  - All bits are used to represent the magnitude of the value
  - The dominant representation for binary arithmetic

50 → 0011 0010
-50 → 1100 1110

-37 →
10010110_b 2's comp →

Number Systems

- Two's Complement

convert -37 decimal to two's complement

8 bits → positive bit values of x | 64 | 32 | 16 | 8 | 4 | 2 | 1

s = negative → flip all bits and add 1 at end

|-37| = 37

greatest bit value ≤ 37 = 32 — 0 0 1

37 – 32 = 5

greatest bit value ≤ 5 = 4 — 0 0 1 0 0 1

5 – 4 = 1

greatest bit value ≤ 1 = 1 — 0 0 1 0 0 1 0 1

1 – 1 = 0

Number Systems

- Two's Complement

convert -37 decimal to two's complement – cont'd

s = negative → flip all bits and add 1 at end

flip +1

0 0 1 0 0 1 0 1 → 1 1 0 1 1 0 1 0 → 1 1 0 1 1 0 1 1

-37 → 1 1 0 1 1 0 1 1 two's complement

Number Systems

- Two's Complement

convert 10010110 two's complement to decimal

MSB is 1 (negative) → remember this for the end
→ flip the bits and add 1 (works both directions)

10010110 $\xrightarrow[\text{flip}]{}$ 01101001 $\xrightarrow[+1]{}$ 01101010

8 bits → positive bit values of x | 64 | 32 | 16 | 8 | 4 | 2 | 1

1*64 + 1*32 + 0*16 + 1*8 + 0*4 + 1*2 + 0*1
64 + 32 + 8 + 2 = 106

sign = 1 = negative → -106

10010110_b 2's comp → -106

Number Systems

- Two's Complement
  - Maximum values:
    - 4 bits = +7, -8 = $2^3-1, -2^3$
    - 8 bits = + 127, -128 = $2^7-1, -2^7$
    - 16 bits = + 32,767, -32,768 = $2^{15}-1, -2^{15}$
  - Not Symmetric

7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8
0111	0110	0101	0100	0011	0010	0001	0000	1111	1110	1101	1100	1011	1010	1001	1000

Number Systems

- Two's Complement
  - Advantages
    - Addition is done the same way as unsigned numbers – same adder circuit
    - ONLY 1 ZERO !
    - Simple word length extension
  - Disadvantages
    - Asymmetric range
    - Harder to do comparisons
    - Not intuitive

7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8
0111	0110	0101	0100	0011	0010	0001	0000	1111	1110	1101	1100	1011	1010	1001	1000

Number Systems

- Two's Complement
  - Sign Extension
    - When extending to larger word sizes, extend the MSB to the left

4 bit	8 bit	16 bit
0110 →	00000110 →	0000000000000110
1001 →	11111001 →	1111111111111001

this works for 1's complement also
not the same for signed magnitude: -1 = 1001 → 10000001 = -1

Number Systems

- Two's Complement
  - Fast way to do 2's complement conversions
    - working from the right
      find the first 1 and leave it and all preceding 0's the same
      flip all remaining bits to the left
      remember the MSB value and set the sign

10010110 2's complement

10 - first 1 from the right

01101010 - all remaining bits flipped

106

- 106 - since we started with a MSB = 1 (negative)