Last updated 7/13/20

Simple FSMs

These slides review the basics of optimizing state diagrams

Upon completion: You should be able to use both techniques to optimize a state diagram

- Two formal approaches
 - Successive Partitions
 - Implication Chart

- Redundant / Equivalent States
 - Successive Partitions

State	Input	Next State	
А	0	В	
А	1	С	
В	0	D	
В	1	F	
С	0	F	
С	1	Е	
D	0	В	
D	1	G	
Е	0	F	
E	1	С	
F	0	Е	
F	1	D	
G	0	F	
G	1	G	

State	Output
А	1
В	1
С	0
D	1
E	0
F	0
G	0

State	Input	Next State	Output
А	0	В	1
А	1	С	1
В	0	D	1
В	1	F	1
С	0	F	0
С	1	Е	0
D	0	В	1
D	1	G	1
Е	0	F	0
Е	1	С	0
F	0	Е	0
F	1	D	0
G	0	F	0
G	1	G	0

- Redundant / Equivalent States
 - Successive Partitions

State	Input	Next State	Output	Partitions	Next States	Action
А	0	В	1	DO	ABODEEG	
А	1	С	1	FU	4101000	
В	0	D	1		1101000	
В	1	F	1			
С	0	F	0			
С	1	Е	0		Identify states and outputs	
D	0	В	1			
D	1	G	1			
E	0	F	0			
Е	1	С	0			
F	0	Е	0			
F	1	D	0			
G	0	F	0			
G	1	G	0			
G	1	G	0			

- Redundant / Equivalent States
 - Successive Partitions

State	Input	Next State	Output	Partitions	Next States	Action
A	0	В	1	PO	ABCDEFG	Separate
A	1	С	1		1101000	
В	0	D	1			ABD, CEFG
В	1	F	1			
С	0	F	0			
С	1	E	0		Partition into sets with same outputs	
D	0	В	1			
D	1	G	1			
E	0	F	0			
Е	1	С	0			
F	0	E	0			
F	1	D	0			
G	0	F	0			
G	1	G	0			

- Redundant / Equivalent States
 - Successive Partitions

Partitions	Next S	Action	
P0	ABCDEFG 1101000		Separate ABD, CEFG
P1	ABD	CEFG	
ln = 0	BDB	FFEF	
ln = 1	CFG	ECDG	

Identify next states based on current state and inputs

State

А

A B

В

С

С

D

D

F

G

G

Input

0

1

0

1

0

1

0

1

0

1

0

1

0

1

Next

State

В

С

D

F

F

Е

В

G

F

С

Е

D

G

Out

1

0

0

0

0

0

0

- Redundant / Equivalent States
 - Successive Partitions

[#] Par	rtitions	Next S	tatos			
			Next States			
PO		ABCD 1101	Separate ABD, CEFG			
P1 In = In =	= 0 = 1	ABD BDB CFG	CEFG FFEF ECDG	Separate CEG and F		

Identify any groups of next states that are not part of an existing partition

Separate those groups by current state

State

А

A B

В

С

С

D

D

E

F

F G

G

Input

0

1

0

1

0

1

0

1

0

1

0

1

0

1

Next State

В

С

D

F

F

Е

В

G

F

С

Е

D

G

0

0

0

0

0

- Redundant / Equivalent States
 - Successive Partitions

Output	Partitions	Next S	Action		
1 1 1	PO	ABCD 1101	Separate ABD, CEFG		
1 0 0 1 1	P1 In = 0 In = 1	ABD BDB CFG	<mark>CEFG</mark> FFEF ECDG		Separate CEG and F
	P2 In = 0 In = 1	ABD BDB CFG	CEG F FFF E ECG D		

Identify next states based on current state and inputs

State

А

A B

В

С

С

D

D

E

F

F G

G

Input

0

1

0

1

0

1

0

1

0

1

0

1

0

1

Next State

В

С

D

F

F

Е

В

G

F

С

Е

D

F

G

0

- Redundant / Equivalent States
 - Successive Partitions

ext ate	Output	Partitions	Next S	Action		
3 C D	1 1 1	P0	ABCD 1101	Separate ABD, CEFG		
= = = 3 5	1 0 0 1 1	P1 In = 0 In = 1	ABD BDB CFG	CEFC FFEF ECDC	Separate CEG and F	
: 2 : : :	0 0 0 0	P2 In = 0 In = 1	ABD BDB CFG	CEG FFF ECG	F E D	Separate AD and B

Identify any groups of next states that are not part of an existing partition

Separate those groups by current state

State

A A

В

В

c c

D

D

G

Input

0

1

0

1 0

1

0

1

G

0

St

- Redundant / Equivalent States
 - Successive Partitions

Output	Partitions		Next States			
1 1 1	PO		ABCD 1101	EFG 000		Separate ABD, CEFG
1 0 0 1 1	P1 In = 0 In = 1	A B C	<mark>BD</mark> DB FG	CEFC FFEI ECDC	G	Separate CEG and F
0 0 0 0	P2 In = 0 In = 1	A B C	ABD BDB CFG		F E D	Separate AD and B
0	P2 In = 0 In = 1	AD BB CG	B D F	CEG FFF ECG	F E D	

Identify next states based on current state and inputs

State

А

А

В

В

С

С

D

D

E

F

F

G

G

Input

0

1

0

1

0

1

0

0

1

0

1

0

1

Next State

В

С

D

F

F

В

G

F

С

E D

F

G

- Redundant / Equivalent States
 - Successive Partitions

Output

1

1

1 1

0

0

1

1

0

0

0

0

0

Partitio	ns		Next S	tates		Action
P0			ABCDEFG 1101000			Separate ABD, CEFG
P1 In = 0 In = 1 Identify any groups of next states that are not part of an existing partition					t ^{eparate} EG and F	
P2		Separate t	hose group	s by current	t state	
In = 0 In = 1		В С	db FG	FFF ECG	E D	Separate AD and B
P2 In = 0 In = 1		AD BB CG	B D F	CEG FFF ECG	F E D	No more reduction

State

А

A B

В

С

С

D

D

E

E F

F

G

G

Input

0

1

0

1

0

1

0

1

0

1

0

1

0

1

Next

State

В

С

D

F

F

Е

В

G

F

С

Е

D

F

G

- Redundant / Equivalent States
 - Successive Partitions

Output	Partitions		Next S	tates		Action
1 1 1	P0		ABCD 1101	EFG 000		Separate ABD, CEFG
1 0 0 1 1	P1 In = 0 In = 1	ABD BDB CFG		CEFG FFEF ECDG		Separate CEG and F
0 0 0 0	P2 In = 0 In = 1	A B C	<mark>BD</mark> DB FG	CEG FFF ECG	F E D	Separate AD and B
0	P2 In = 0 In = 1	AD BB CG	B D F	CEG FFF ECG	F E D	No more reduction
	Pfinal	AD	В	CEG	F	

State

А

A B

В

С

С

D

D

E

E F

F

G

G

Input

0

1

0

1 0

1

0

1

0

1

0

1

0

1

Next State

В

С

D F

F

Е

В

G

F

С

Е

D

F

G

	• P(adur	dant /	Fo		lon	+ 5+-	to	Partitions		Ne	ext State	S		Action
								ite	РО		/	ABCDEFG 1101000			Separate ABD, CEFG
	•	Succe	essive Pa	rti	tions				P1 In = 0		ABD BDB		CEFG FFEF		Separate
State	Input	Next State	Output						in = 1 P2		ABD	(ECDG	F	CEG and F
А	0	В	1		7 sta	tes \rightarrow	4 stat	es	ln = 0 ln = 1		BDB CFG	I	FFF ECG	E D	Separate AD and B
А	1	С	1						P2	AD	В	(CEG	F	No more
В	0	D	1						ln = 1	CG	F	ł	ECG	D	reduction
В	1	F	1						Pfinal	AD	В	(CEG	F	
С	0	F	0		State	Input	Next		Output		State	Input	Next		Output
С	1	Е	0				State						State	2	
D	0	В	1		AD	0	В		1		Р	0	Q		1
D	1	G	1	~	AD	1	CEG		1	D	Р	1	R		1
E	0	F	0	4	В	0	AD		1	7	Q	0	Р		1
Е	1	С	0		В	1	F		1		Q	1	S		1
F	0	Е	0		CEG	0	F		0		R	0	S		0
F	1	- D	0		CEG	1	CEG		0		R	1	R		0
Ģ	1	F	0		F	0	CEG		0		S	0	R		0
G	1	Г	0		F	1	AD		0		S	1	Р		0
G 1911	T	G	U				14								(C) †
							and a second sec								Gi

- Redundant / Equivalent States
 - Implication Chart

State	Input	Next State	Output
А	0	В	1
А	1	С	1
В	0	D	1
В	1	F	1
С	0	F	0
С	1	Е	0
D	0	В	1
D	1	G	1
Е	0	F	0
Е	1	С	0
F	0	Е	0
F	1	D	0
G	0	F	0
G	1	G	0



- Redundant / Equivalent States
 - Implication Chart

State	Input	Next State	Output	
А	0	В	1	
А	1	С	1	
В	0	D	1	
В	1	F	1	
С	0	F	0	
С	1	Е	0	
D	0	В	1	
D	1	G	1	
Е	0	F	0	
Е	1	С	0	
F	0	Е	0	
F	1	D	0	
G	0	F	0	
G	1	G	0	

compare pairs of states If outputs are different X out box



- Redundant / Equivalent States
 - Implication Chart

State	Input	Next State	Output	
А	0	B	1	
А	1	C	1	N
В	0	D	1	\mathcal{I}
В	1	F	1	
С	0	F	0	1
С	1	Е	0	
D	0	В	1	
D	1	G	1	
Е	0	F	0	
Е	1	С	0	
F	0	Е	0	
F	1	D	0	
G	0	F	0	
G	1	G	0	

write in the implicants to all empty boxes The two states would be the same IFF the implicants are the same



- Redundant / Equivalent States
 - Implication Chart

State	Input	Next State	Output	
А	0	В	1	
А	1	С	1	N
В	0	D	1	
В	1	F	1	
С	0	F	0	/
С	1	Е	0	
D	0	В	1	
D	1	G	1	
Е	0	F	0	
E	1	С	0	
F	0	Е	0	
F	1	D	0	
G	0	F	0	
G	1	G	0	

Traverse the structure and X out any boxes whose implicants are already X'd out This indicates the implicant is not true



- Redundant / Equivalent States
 - Implication Chart

State	Input	Next State	Output	
А	0	В	1	
А	1	С	1	N
В	0	D	1	\mathcal{F}
В	1	F	1	
С	0	F	0	1
С	1	Е	0	
D	0	В	1	
D	1	G	1	
Е	0	F	0	
E	1	С	0	
F	0	Е	0	
F	1	D	0	
G	0	F	0	
G	1	G	0	

Traverse the structure and X out any boxes whose implicants are already X'd out This indicates the implicant is not true



- Redundant / Equivalent States
 - Implication Chart

State	Input	Next State	Output	
А	0	В	1	
А	1	С	1	1
В	0	D	1)
В	1	F	1	1
С	0	F	0	1
С	1	Е	0	
D	0	В	1	
D	1	G	1	
Е	0	F	0	
Е	1	С	0	
F	0	Е	0	
F	1	D	0	
G	0	F	0	
G	1	G	0	

Traverse the structure and X out any boxes whose implicants are already X'd out This indicates the implicant is not true



- Redundant / Equivalent States
 - Implication Chart

State	Input	Next State	Output
А	0	В	1
А	1	С	1
В	0	D	1
В	1	F	1
С	0	F	0
С	1	Е	0
D	0	В	1
D	1	G	1
Е	0	F	0
Е	1	С	0
F	0	Е	0
F	1	D	0
G	0	F	0
G	1	G	0

Remaining un-X'd boxes indicate equivalent states



- Redundant / Equivalent States
 - Implication Chart

State	Input	Next State	Output
А	0	В	1
А	1	С	1
В	0	D	1
В	1	F	1
С	0	F	0
С	1	Е	0
D	0	В	1
D	1	G	1
Е	0	F	0
Е	1	С	0
F	0	Е	0
F	1	D	0
G	0	F	0
G	1	G	0

Any duplicate entries indicate independent states



- Redundant / Equivalent States
 - Implication Chart

State	Input	Next State	Output
А	0	В	1
А	1	С	1
В	0	D	1
В	1	F	1
С	0	F	0
С	1	Е	0
D	0	В	1
D	1	G	1
Е	0	F	0
Е	1	С	0
F	0	Е	0
F	1	D	0
G	0	F	0
G	1	G	0



- Redundant / Equivalent States
 - Implication Chart

State	Input	Next State	Output	
А	0	В	1	
А	1	С	1	
В	0	D	1	
В	1	F	1	
С	0	F	0	
С	1	E	0	
D	0	В	1	
D	1	G	1	ひ
Е	0	F	0	
E	1	С	0	
F	0	Е	0	
F	1	D	0	
G	0	F	0	
G	1	G	0	

7 states \rightarrow 4 states

State	Input	Next State	Output
AD	0	В	1
AD	1	CEG	1
В	0	AD	1
В	1	F	1
CEG	0	F	0
CEG	1	CEG	0
F	0	CEG	0
F	1	AD	0



	State	Input	Next State	Output
	Р	0	Q	1
>	Р	1	R	1
	Q	0	Р	1
	Q	1	S	1
	R	0	S	0
	R	1	R	0
	S	0	R	0
	S	1	Р	0

Optimized FSMs

Design Process Circuit Design

- 1) Identify the states collectively these make a state variable
- 2) Identify the Inputs and Outputs
- 3) Assign values for each input/output (encoding)
- 4) Create a state transition diagram / table
- 5) Optimize the state transition table
- 6) Assign values for the state variable for each state (encoding)
- 7) Create truth tables for the combinational logic blocks in the machine model: next state, output
- 8) Minimize the next state and output equations using K-maps or Boolean Algebra techniques
- 9) Draw the circuit schematic
- 10) Verify the solution
- 11) Build the physical circuit
- 12) Test the physical circuit to ensure correct operation

Design Process HDL

- 1) Identify the states collectively these make a state variable
- 2) Identify the Inputs and Outputs
- 3) Create a state transition diagram / table
- 4) Optimize the state transition table
- 5) Create the HDL to match the state transition table
- 6) Choose an encoding scheme (or let the tool decide)
- 7) Synthesize the design
- 8) Verify the solution
- 9) Build the physical circuit
- 10) Test the physical circuit to ensure correct operation