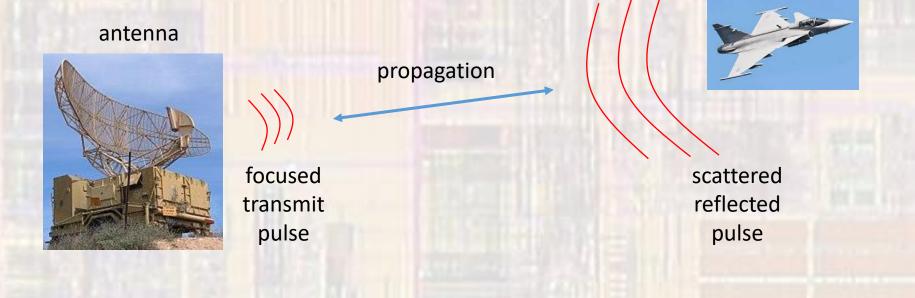
## Last updated 4/21/20

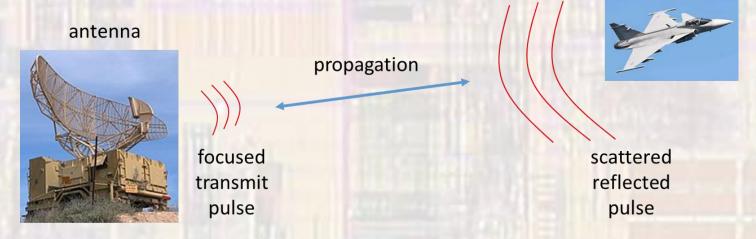
- RADAR
  - RAdio Detection And Ranging
  - Early development in 1936
  - Significant advancements during WWII
    - Detect incoming bombers
    - Prevented Germany from achieving air superiority
- Key advantages
  - Long range
  - All weather
  - 3D target position detection
  - Can be mobile
  - Many variations achievable

- Basic idea
  - Transmit a focused pulsed radio wave
  - Detect the returning radio wave
  - Direction of transmit and time of arrival of the returned signal → location

target (w/ cross section)



- Basic idea
  - Detect range
  - Detect azimuth (angle wrt north)
  - **Detect elevation**
  - Detect size (cross section)
  - Detect speed (using doppler)



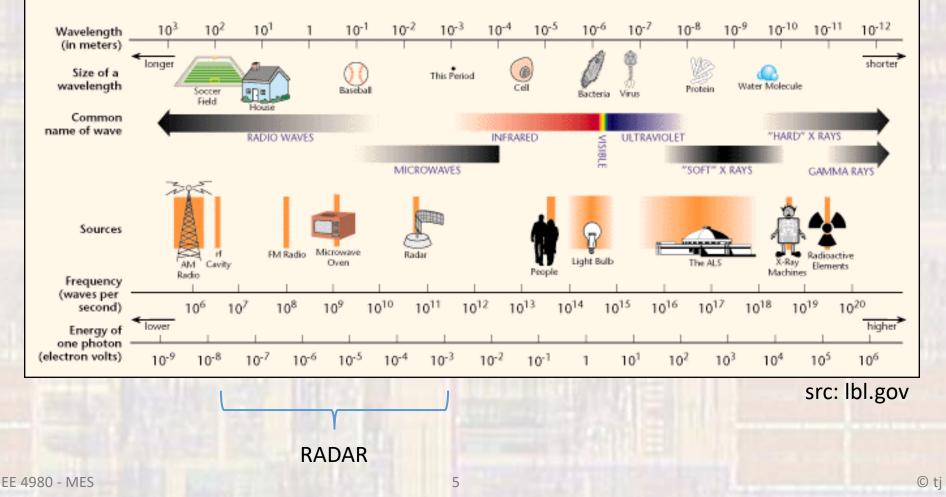
target (w/ cross section)



EE 4980 - MES

RADAR frequencies

#### THE ELECTROMAGNETIC SPECTRUM



#### RADAR frequency bands



Frequency Range	Wavelength Range	Band Name	Usage
3-30 MHz	10-100 m	HF	Coastal radar systems
30-300 MHz	1-10 m	VHF	Very long range
300-1000 MHz	0.3-1 m	UHF	Very long range
1-2 GHz	15-30 cm	L-band	Long range
2-4 GHz	7.5-15 cm	S-band	Terminal air traffic control, marine radar
4-8 GHz	3.75-7.5 cm	C-band	Satellite transponders, synthetic aperture radar
8-12 GHz	2.5-3.75 cm	X-band	Marine radar, weather, ground surveillance, synthetic aperture radar
12-18 GHz	1.67-2.5 cm	Ku-band	Satellite transponders
18-24 GHz	1.11-1.67 cm	K-band	Satellite transponders, radar guns, weather
24-40 GHz	0.75-1.11 cm	Ka-band	Mapping, surveillance

- RADAR equation
  - Trivial version

range = Ct/2

 $C = Speed of light = 3x10^8 m/s$ 

#### t = time between transmit and return

antenna propogation focused transmit pulse (w/ cross section) focused transmit pulse

target

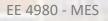
- RADAR equation
  - Transmitter power
  - P<sub>T</sub> = Peak transmit power
  - P<sub>AVE</sub> = Average value
    - Transmitting pulses
    - Duty cycle = pulse width / pulse repetition interval

Pulse width

- P<sub>AVE</sub> = P<sub>T</sub> \* Duty Cycle
- Ex.
  - 100us pulse with a 1MW peak power
  - 1ms pulse repetition interval (1Kz pulse frequency)

8

- $\rightarrow$  10% duty cycle
- $\rightarrow$  100KW average transmit power



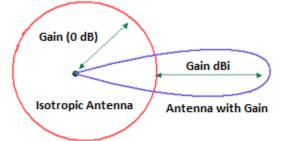
1ms

100us

pulsed signal

- RADAR equation
  - Transmit Gain
    - Use directional antennas for transmit

$$G = \frac{4\pi A}{\lambda^2}$$



src: everything RF

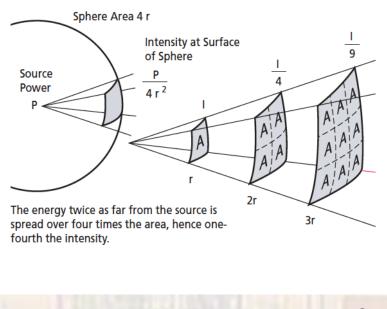
- A = Antenna aperture (effective aperture)
- λ = pulse signal wavelength

= CT = C/f

- RADAR equation
  - Transmit signal spread factor

$$S_{Tx} = \frac{1}{4\pi R^2}$$

• R = range



src: NASA/JPL

- RADAR equation
  - Transmit losses
    - Signal generation losses
    - Antenna losses

L

- Environmental losses (atmospheric)
- Generally lumped together into a single factor

- RADAR equation
  - Signal power density at the target

# $P_{D_{target}} = [P_T] \begin{bmatrix} 4\pi A \\ \lambda^2 \end{bmatrix} \begin{bmatrix} 1 \\ 4\pi R^2 \end{bmatrix} \begin{bmatrix} 1 \\ L \end{bmatrix}$

- RADAR equation
  - Radar cross section RCS
    - σ
    - Apparent size of target
    - No necessarily the actual size , but a measure of how much of the incident radiation it reflects
    - m<sup>2</sup>
    - Front of a truck vs the front to a sports car
    - smooth surface vs a concave space

- RADAR equation
  - Reflected power
    - Incident wave power density x radar cross section

• 
$$P_{Reflected} = [P_T] \left[ \frac{4\pi A}{\lambda^2} \right] \left[ \frac{1}{4\pi R^2} \right] \left[ \frac{1}{L} \right] \sigma$$

- RADAR equation
  - Receive signal spread factor

$$S_{Rx} = \frac{1}{4\pi R^2}$$

• R = range

RADAR equation

A

- Receive Aperture
  - Measure of how effective an antenna is at receiving the power of specific electromagnetic radiation

- RADAR equation
  - Dwell Time

τ

- The time that an antenna beam spends on a target
- Dependent on the beam size and speed of rotation of the antenna

RADAR equation

*Receive Signal Energy* =

$$\begin{bmatrix} P_T \end{bmatrix} \begin{bmatrix} \frac{4\pi A}{\lambda^2} \end{bmatrix} \begin{bmatrix} \frac{1}{4\pi R^2} \end{bmatrix} \begin{bmatrix} \frac{1}{L} \end{bmatrix} \begin{bmatrix} \sigma \end{bmatrix} \begin{bmatrix} \frac{1}{4\pi R^2} \end{bmatrix} \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} \tau \end{bmatrix}$$
$$W \qquad \frac{1}{m^2} \qquad m^2 \quad \frac{1}{m^2} \qquad m^2 \quad s = Ws$$

- RADAR equation
  - Noise
    - Atmospheric interference
    - Solar noise
    - Ground noise
    - Other EM noise
    - System noise
    - Assume Noise can be characterized as a noise temperature = T<sub>s</sub>

#### Noise power(N) = $kB_NT_S$

- k Boltzmann's constant =  $1.38 \times 10^{-23}$  joules / K
- B<sub>N</sub> receiver noise bandwidth

#### RADAR equation

- Signal to Noise Ratio Tracking version
  - Know where the target is → dwell time not part of the analysis
  - S/N = Received signal power / Noise power

$$[P_T] \left[ \frac{4\pi A}{\lambda^2} \right] \left[ \frac{1}{4\pi R^2} \right] \left[ \frac{1}{L} \right] [\sigma] \left[ \frac{1}{4\pi R^2} \right] [A]$$
$$kB_N T_S$$

Note : 
$$G_T = \begin{bmatrix} \frac{4\pi A}{\lambda^2} \end{bmatrix}$$
  
Let :  $G_R = \begin{bmatrix} \frac{4\pi A}{\lambda^2} \end{bmatrix}$  [A]  $\rightarrow \begin{bmatrix} \frac{G_R \lambda^2}{4\pi} \end{bmatrix}$ 

Assume G<sub>T</sub> = G<sub>R</sub> = G

$$S/_N = \frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_S B_N L}$$

- RADAR equation
  - Signal to Noise Ratio Searching version
    - Need to scan for the target
      - Average power = P<sub>AV</sub>
      - Solid Angle =  $\Omega$
      - Scan time for  $\Omega = t_s$

 $\frac{S}{N} = \frac{P_{av}At_s\sigma}{4\pi\Omega R^4 kT_s L}$ 

- RADAR equation
  - Signal to Noise Ratio Searching version

$$S/_{N} = \frac{P_{av}At_{s}\sigma}{4\pi\Omega R^{4}kT_{s}L}$$

solving for P<sub>av</sub>

$$P_{av} = \frac{4\pi\Omega R^4 k T_S L\left(S/N\right)}{A t_s \sigma}$$

- Linear function of everything except R
- Strong function of R

- RADAR equation
  - P<sub>av</sub> Searching version

$$P_{av} = \frac{4\pi\Omega R^4 k T_S L\left(\frac{S}{N}\right)}{At_s \sigma}$$

- Assuming a given RADAR system performance and hardware:
  - doubling the search range requires a 16x increase in the average power
  - capturing a ½ size target with the same s/n requires a 2x increase in average power

- RADAR equation ex. Airport Tracking RADAR
  - RADAR parameters

RADAR SYSTEM PARAMETERS				
Peak Power	1.4 MW	61.46 dB		
Antenna Aperture	5 m x 3 m			
Pulsed Signal Frequency	2.8 GHz			
Pulse Signal Wavelength	103 mm	-9.9 dB		
Pulse Width	600 ns			
Pulse Repetition Rate	1200 Hz			
Pulse Duty Cycle	0.00072%			
Receiver Noise Bandwidth	1.5 MHz	61.7 dB		
Effective Noise Temperature	900 K	29.5 dB		
Typical system Losses	6.3	8 dB		
Antenna Rotation Rate	12 rpm			
Azimuth <mark>Beamwid</mark> th	1.3 °			
Calculated Parameters				
Antenna Gain $4\pi A / \lambda 2$	17,767	42 dB		
with beam forming losse	es 1,700	32.3 dB		
Pulses per Beamwidth	22.5			
Constants				
4pi	12.566	11 dB		
k	1.38E-23	-228.6 dB		

- RADAR equation ex. Airport Tracking RADAR
  - RADAR parameters

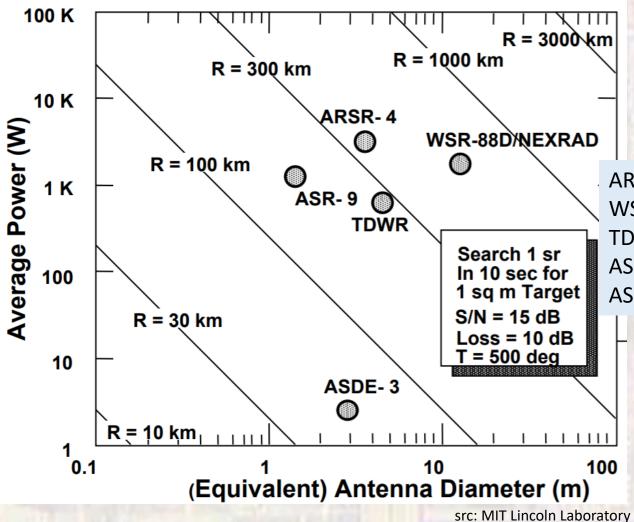
$$S/_{N} = \frac{P_{T}G^{2}\lambda^{2}\sigma}{(4\pi)^{3}R^{4}kT_{S}B_{N}L}$$

$$S/_{N} per pulse = 1.84 \ge 10^{20} \frac{\sigma}{R^4} = 202.6 dB \frac{\sigma}{R^4}$$

- given a 1m<sup>2</sup> target at 70miles
  - S/N per pulse = 1.14
  - S/N per dwell = 25.7

= 0.569dB = 14.09dB

RADAR performance



ARSR: Air Route Surveillance WSR-88D: Weather TDWR: Doppler Weather ASR-9: Airport Surveillance ASDE-3: Airport Surface

#### RADAR performance

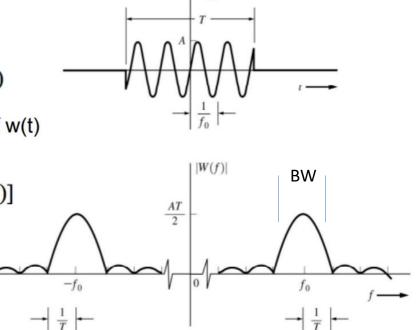
Waveform of a switch sinusoid can be represented as follow:

$$\omega(t) = \Pi(\frac{t}{T})A\sin\omega_0 t = \Pi(\frac{t}{T})A\cos(\omega_0 t - \frac{\pi}{2})$$

The frequency domain representation of w(t) will be:

$$W(f) = j \frac{A}{2} T[Sa(\pi T(f + f_0)) - Sa(\pi T(f - f_0))]$$

Note that the spectrum of w(t) is imaginary!



w(t)

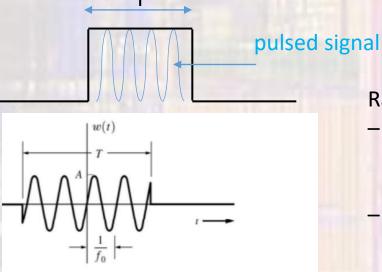
As  $T \rightarrow INF$ ,  $1/T \rightarrow 0$ , then Sa waveform converges to a double-sided delta waveform

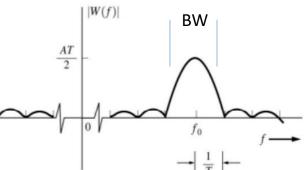
#### Magnitude Spectrum of w(t)

src: Middle Tennessee State University un-attributed

#### RADAR performance

Pulse width





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Range Resolution  $\Delta r$ – Proportional to pulse width (T)  $\Delta r = (CT)/2$ 

- Inversely proportional to bandwidth (B = 1/T)  $\Delta r = C/(2B)$ 

1 MHz Bandwidth =>  $\Delta r$  = 150 m of range error

RADAR performance

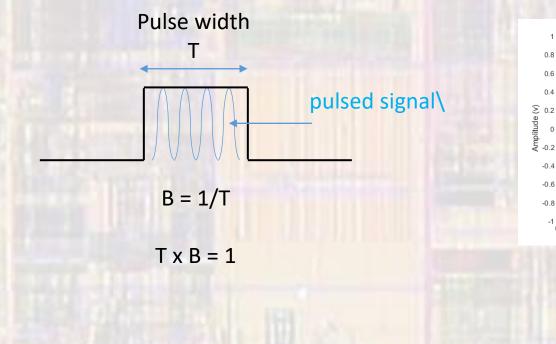
 $\Delta r = (CT)/2 = C/(2B)$ 

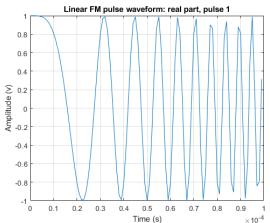
 $S/_{N} = \frac{P_{av}At_{s}\sigma}{4\pi\Omega R^{4}kT_{s}L}$ 

- P<sub>av</sub> ~ pulse width T
- $\Delta r \sim pulse width T, \sim 1/B$
- Difficult to get good S/N and good range error at the same time

- RADAR performance
  - 2 approaches to get: high average power (wide T) and low range error (high B)
    - Frequency modulated carrier
    - Phase modulated carrier

- RADAR performance
  - Frequency modulated pulse Linear

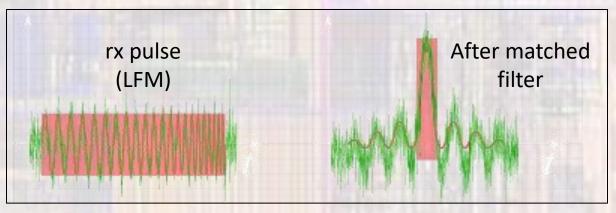




 $B = \Delta F = F2 - F1$ 

 $T \times B = T C$ 

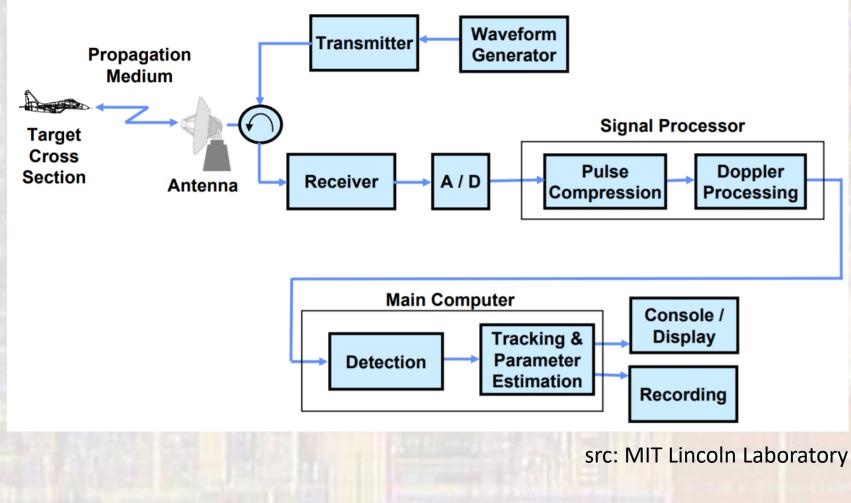
- RADAR performance
  - Frequency modulated pulse Linear



src: radartutorial.eu

- Fix T  $\rightarrow$  P<sub>av</sub>  $\rightarrow$  good S/N
- Linear frequency Modulation  $\rightarrow \uparrow B \rightarrow \downarrow \Delta r$
- Requires a matched filter in receiver

RADAR System



RADAR Mitigation

