RADAR Equation

Last updated 3/27/24

RADAR Equation

- RADAR equation

  - Signal to Noise Ratio - Tracking version

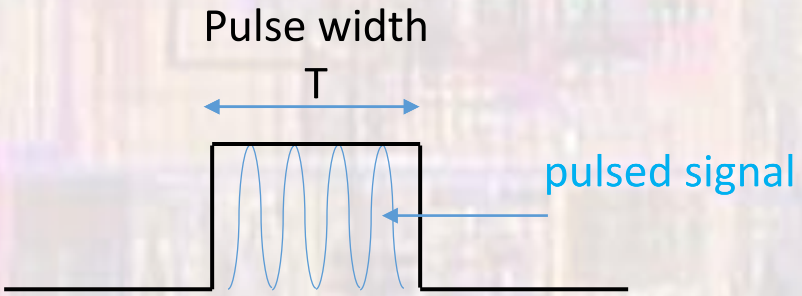
$$S/_N = \frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_S B_N L}$$

  - Signal to Noise Ratio - Searching version

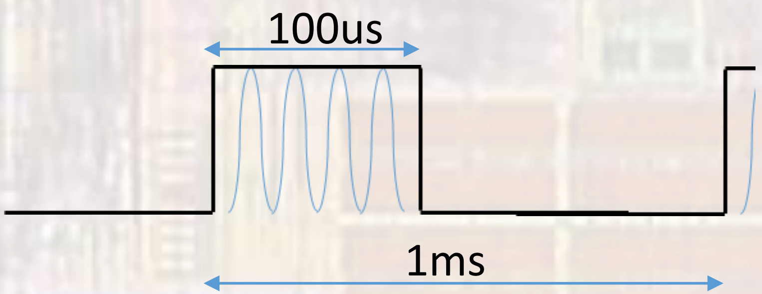
$$S/_N = \frac{P_{av} A t_s \sigma}{4\pi \Omega R^4 k T_S L}$$

RADAR Equation

- Transmitter power
  - P_T = Peak transmit power
  - P_{AVE} = Average value
    - Transmitting pulses
    - Duty cycle = pulse width / pulse repetition interval
    - $P_{AVE} = P_T$ * Duty Cycle

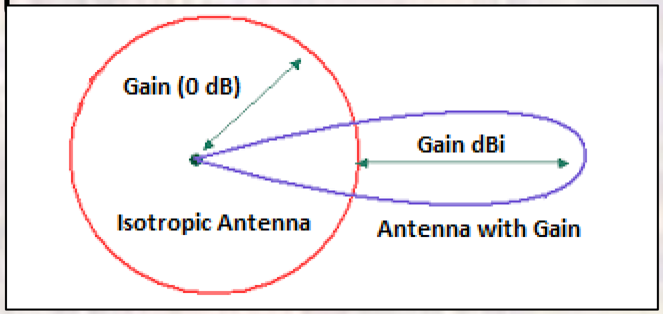


- Ex.
  - 100us pulse with a 1MW peak power
  - 1ms pulse repetition interval (1Kz pulse frequency)
  - →10% duty cycle
  - → 100KW average transmit power



RADAR Equation

- Transmit Gain
  - Use directional antennas for transmit

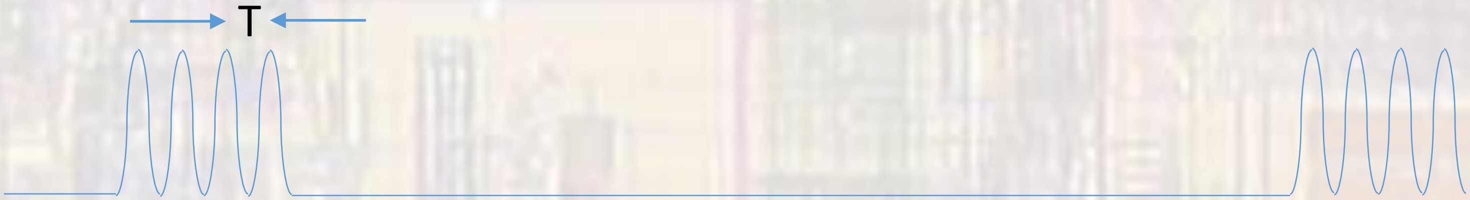


src: everything RF

$$G = \frac{4\pi A}{\lambda^2}$$

  - A = Antenna aperture (effective aperture)
  - λ = pulse signal wavelength

    = CT = C/f

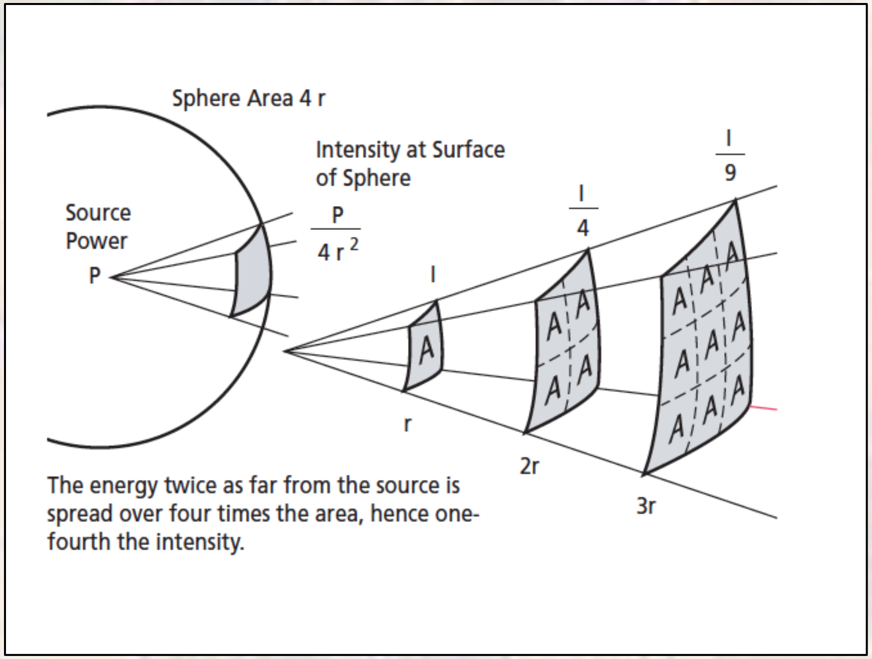


RADAR Equation

- Transmit signal spread factor

$$S_{Tx} = \frac{1}{4\pi R^2}$$

  - R = range



src: NASA/JPL

RADAR Equation

- Transmit losses
  - Signal generation losses
  - Antenna losses
  - Environmental losses (atmospheric)
  - Generally lumped together into a single factor

$$L$$

RADAR Equation

- Signal power density at the target

$$P_{D_{target}} = [P_T]\left[\frac{4\pi A}{\lambda^2}\right]\left[\frac{1}{4\pi R^2}\right]\left[\frac{1}{L}\right]$$

RADAR Equation

- Radar cross section – RCS
  - σ
    - Apparent size of target
    - No necessarily the actual size , but a measure of how much of the incident radiation it reflects
    - m^2
  - Front of a truck vs the front to a sports car
  - smooth surface vs a concave space

RADAR Equation

- Reflected power
  - Incident wave power density X radar cross section

- $P_{Reflected} = [P_T]\left[\frac{4\pi A}{\lambda^2}\right]\left[\frac{1}{4\pi R^2}\right]\left[\frac{1}{L}\right]\sigma$

RADAR Equation

- Receive signal spread factor

$$S_{Rx} = \frac{1}{4\pi R^2}$$

  - R = range

RADAR Equation

- Receive Aperture
  - Measure of how effective an antenna is at receiving the power of specific electromagnetic radiation

$$A$$

RADAR Equation

- Dwell Time
  - The time that an antenna beam spends on a target
  - Dependent on the beam size and speed of rotation of the antenna

$$\tau$$

RADAR Equation

$$Receive\ Signal\ Energy =$$

$$[P_T]\left[\frac{4\pi A}{\lambda^2}\right]\left[\frac{1}{4\pi R^2}\right]\left[\frac{1}{L}\right][\sigma]\left[\frac{1}{4\pi R^2}\right][A][\tau\,]$$

$$W \qquad \frac{1}{m^2} \qquad m^2 \quad \frac{1}{m^2} \quad m^2 \;\; s = Ws$$

RADAR Equation

- Noise
  - Atmospheric interference
  - Solar noise
  - Ground noise
  - Other EM noise
  - System noise
  - Assume – Noise can be characterized as a noise temperature = T_S

$$\text{Noise power}(N) = kB_NT_S$$

- k – Boltzmann's constant = 1.38×10^{-23} joules / K
- B_N – receiver noise bandwidth

RADAR Equation

- Signal to Noise Ratio - Tracking version
  - Know where the target is → dwell time not part of the analysis
  - S/N = Received signal power / Noise power

$$\frac{[P_T]\left[\frac{4\pi A}{\lambda^2}\right]\left[\frac{1}{4\pi R^2}\right]\left[\frac{1}{L}\right][\sigma]\left[\frac{1}{4\pi R^2}\right][A]}{kB_NT_S}$$

- Note : $G_T = \left[\frac{4\pi A}{\lambda^2}\right]$
- Let : $G_R = \left[\frac{4\pi A}{\lambda^2}\right]$ $[A] \rightarrow \left[\frac{G_R\lambda^2}{4\pi}\right]$
- Assume $G_T = G_R = G$

$$S/_N = \frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_S B_N L}$$

RADAR Equation

- Signal to Noise Ratio - Searching version
  - Need to scan for the target
    - Average power = P_{AV}
    - Solid Angle = Ω
    - Scan time for Ω = t_S

$$S/_N = \frac{P_{av} A t_s \sigma}{4\pi\Omega R^4 k T_S L}$$

RADAR Equation

- Signal to Noise Ratio - Searching version

$$S/_N = \frac{P_{av} A t_s \sigma}{4\pi\Omega R^4 k T_S L}$$

solving for P_{av}

$$P_{av} = \frac{4\pi\Omega R^4 k T_S L \left(S/_N\right)}{A t_s \sigma}$$

- Linear function of everything except R
- Strong function of R

RADAR Equation

- P_{av} - Searching version

$$P_{av} = \frac{4\pi\Omega R^4 k T_S L\left({}^{S}/_{N}\right)}{A t_s \sigma}$$

- Assuming a given RADAR system performance and hardware:
  - doubling the search range requires a 16x increase in the average power
  - capturing a ½ size target with the same s/n requires a 2x increase in average power