RC Active Filters

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RC Active Filters

- Passive filter concerns
  - Each stage loads the previous stage
  - Best case gain is 1 (0dB)
  - Any non-infinite output load will change the filter output

\[
\begin{align*}
\text{Cascaded Filter} & \\
+ & - \\
V_{\text{in}} & \text{R}_{\text{load}} & V_{\text{out}}
\end{align*}
\]
RC Active Filters

• Why RC
  • It is much easier and cheaper to build integrated circuit capacitors and resistors than inductors
RC Active Filters

- Just buffer the passive filter – Low Pass
  - Non-inverting
  - Load insensitive
  - Can be cascaded
  - Unity Gain

\[ f_c = \frac{1}{2\pi RC} \]

\[ A = 1 \]

\[ A_v = \frac{A}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}} \]
RC Active Filters

- Just buffer the passive filter – **Caveat # 1**
  - The OpAmp has an internal Lowpass characteristic (GBWP)
    - Can be good or bad depending on the situation
RC Active Filters

- Buffer the passive filter with gain – Low Pass
  - Non-inverting
  - Load insensitive
  - Can be cascaded
  - Selectable Gain

\[ A = 1 + \frac{R_F}{R_I} \]

\[ f_C = \frac{1}{2\pi RC} \]

\[ A_v = \sqrt{1 + \left(\frac{f}{f_C}\right)^2} \]
RC Active Filters

• Buffer the passive filter – Caveat # 2
  • The filter has a relatively small input impedance
    • Loads the driver
    • Driver output impedance may affect the filter
RC Active Filters

- Buffer the passive filter – Low Pass
  - Remove the reactive element from the input
  - Inverting
  - Selectable Gain

\[
A = -\frac{R_F}{R_I}
\]

\[
f_C = \frac{1}{2\pi R_F C}
\]

\[
A_v = \sqrt{1 + \left(\frac{f}{f_C}\right)^2}
\]
RC Active Filters

• Buffer the passive filter – Low Pass
  • Remove the reactive element from the input
  • Non-inverting
  • Selectable gain
  • High frequencies $\Rightarrow$ unity gain

$$f_C = \frac{1}{2\pi R_F C}$$

$$A = 1 + \frac{R_F}{R_I}$$

$$A_v^* = \frac{A}{\sqrt{1 + \left(\frac{f}{f_C}\right)^2}}$$

* $A_{v \text{ min}} = 1 = 0dB$
1st Order RC Active Filters

- First order – High Pass
  - Non-inverting
  - Unity Gain

\[ A = 1 \]
\[ A_v = \frac{A \left(\frac{f}{f_c}\right)}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}} \]

\[ f_c = \frac{1}{2\pi RC} \]

- Passive Filter

![Passive Filter Diagram]
1\textsuperscript{st} Order RC Active Filters

- First order – High Pass
  - Non-inverting
  - Selectable Gain

\[
A_v = \frac{A \left( \frac{f}{f_C} \right)}{\sqrt{1 + \left( \frac{f}{f_C} \right)^2}}
\]

\[
f_C = \frac{1}{2\pi RC}
\]

\[
A = 1 + \frac{R_F}{R_I}
\]
1\textsuperscript{st} Order RC Active Filters

- First order – High Pass
  - Inverting
  - Selectable Gain

\[ f_C = \frac{1}{2\pi RC} \]
\[ A = -\frac{R_F}{R_I} \]
\[ A_v = \frac{A \left(\frac{f}{f_C}\right)}{\sqrt{1 + \left(\frac{f}{f_C}\right)^2}} \]
1\textsuperscript{st} Order RC Active Filters

- First order – Band Pass
  - Non-inverting
  - Selectable Gain
  - Wide passband

\[ f_{CL} = \frac{1}{2\pi R_2 C_2} \]

\[ f_{CU} = \frac{1}{2\pi R_1 C_1} \]

\[ A = 1 + \frac{R_F}{R_I} \]

\[ A_v = \sqrt{\frac{A \left( \frac{f}{f_{CH}} \right)}{\sqrt{1 + \left( \frac{f}{f_{CL}} \right)^2}}} \]
1st Order RC Active Filters

• First order – Band Pass
  • Inverting
  • Selectable Gain
  • Narrower passbands possible

\[
A_{v}^* = \sqrt{\frac{A}{1 + \left(\frac{f}{f_{CH}}\right)^2}} \sqrt{\frac{1}{1 + \left(\frac{f}{f_{CL}}\right)^2}}
\]

\[
f_{CL} = \frac{1}{2\pi R_1 C_1}
\]

\[
f_{CU} = \frac{1}{2\pi R_2 C_2}
\]

\[
A = -\frac{R_F}{R_I}
\]

* At high frequencies, the OpAmp internal capacitance limits the rolloff
1\textsuperscript{st} Order RC Active Filters

- First order – Band Stop
  - Non-inverting
  - Selectable Gain

\[ A_v^* = \frac{A \left( \frac{f}{f_{CH}} \right)}{\sqrt{1 + \left( \frac{f}{f_{CH}} \right)^2}} \cdot \frac{1}{\sqrt{1 + \left( \frac{f}{f_{CL}} \right)^2}} \]

\[ f_{CL} = \frac{1}{2\pi R_1 C_1} \]

\[ f_{CU} = \frac{1}{2\pi R_2 C_2} \]

\[ A = -\frac{R_F}{R_I} \]

* At high frequencies, the OpAmp internal capacitance limits the rolloff
2\textsuperscript{nd} Order RC Active Filters

- Generalized Second Order Configuration

\[
\begin{align*}
(V_i - V_a) \frac{1}{Z_1} &= (V_a - V_b) \frac{1}{Z_2} - (V_a - V_o) \frac{1}{Z_3} \\
(V_a - V_b) \frac{1}{Z_2} &= V_b \frac{1}{Z_4}
\end{align*}
\]

\[
A_v = \frac{1}{Z_1 Z_2} + \frac{1}{Z_4 \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)}
\]
2\textsuperscript{nd} Order RC Active Filters

- Generalized Second Order Configuration

$$H_s = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

- Can be generalized to the form

$$A_v = \frac{\frac{1}{Z_1} \frac{1}{Z_2}}{\frac{1}{Z_1} \frac{1}{Z_2} + \frac{1}{Z_4} \left( \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right)}$$

- \(a_0 \neq 0\) low pass
- \(a_1 \neq 0\) band pass
- \(a_2 \neq 0\) high pass

\(\omega\) — characteristic frequency
\(Q\) — quality factor
2\textsuperscript{nd} Order RC Active Filters

• Generalized Second Order Configuration

\[ H_S = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \]

• Variations in \( a \), \( \omega \), and \( Q \) generate different passband characteristics

• \( a \), \( \omega \), and \( Q \) are set by the values of \( Z_1, Z_2, Z_3, Z_4 \)
2\textsuperscript{nd} Order RC Active Filters

- Butterworth – Maximally Flat
  - Low Pass

\[ H_s = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \]

For the Butterworth response
\[ R_1 = R_2 \]
\[ C_3 = 2C_4 \]

\[ \omega_{3dB} = \frac{1}{\sqrt{2RC_4}} \]
2\textsuperscript{nd} Order RC Active Filters

- Butterworth – Maximally Flat
  - High Pass

\[ H_s = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \]

For the Butterworth response:

\[ R_4 = 2R_3 \]

\[ C_1 = C_2 \]

\[ \omega_{3dB} = \frac{1}{\sqrt{2R_3C}} \]

\[ f_{3dB} = \frac{1}{2\pi\sqrt{2R_3C}} = 11.23\text{KHz} \]
2nd Order RC Active Filters

- Chebyshev – Enhanced initial rolloff
  - Low Pass

\[ H_s = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \]

Passband ripple
Linear mag plot
2\textsuperscript{nd} Order RC Active Filters

- Second order filters
  - Non-inverting
  - Selectable gain
  - High frequencies \(\rightarrow\) unity gain

\[
A = 1 + \frac{R_F}{R_I}
\]

\[
A_v = A \times \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega_0}{Q} \times \omega\right)^2}}
\]

\[
f_C = \frac{1}{2\pi \sqrt{R_1 C_1 R_2 C_2}}
\]
Second order filters

- 2\textsuperscript{nd} order filters with gain exhibit a more complex response than 2\textsuperscript{nd} order passive filters
- Q – quality factor = gain at the 3db point
- ζ – zeta – damping factor = 1/(2Q)
- \(\omega_0\) – characteristic frequency – where things change in the frequency response

2\textsuperscript{nd} order LPF

\[
A_v = A \times \left| \frac{\omega_0^2}{-\omega^2 + j \left( \frac{\omega_0}{Q} \right) \omega + \omega_0^2} \right| = A \times \frac{\omega_0^2}{\sqrt{\left( \omega_0^2 - \omega^2 \right)^2 + \left( \frac{\omega_0}{Q} \times \omega \right)^2}}
\]

\[
\omega_c = \omega_0 \sqrt{1 - \frac{1}{2Q^2} + \sqrt{1 + \left( 1 - \frac{1}{2Q^2} \right)^2}}
\]

\[
Q = \frac{1}{A - 3}
A = 3 - \frac{1}{Q}
\]
2\textsuperscript{nd} Order RC Active Filters

- Second order filters
  - 2\textsuperscript{nd} order filters with gain exhibit a more complex response than 2\textsuperscript{nd} order passive filters

\[ A_v = A \times \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega_0}{Q} \times \omega\right)^2}} \]

\[ \omega_c = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} + \sqrt{1 + \left(1 - \frac{1}{2Q^2}\right)^2} \]

\[ Q = \frac{1}{\sqrt{2}}, \quad \omega_c = \omega_0, \quad A = 1.58 \]

- \( \omega = 0, \quad A_v = A \)
- \( \omega = \omega_0, \quad A_v = A \times Q \)
- \( \omega \gg \omega_0, \quad A_v = A \times \frac{\omega_0^2}{\omega^2} = 0 \)
2nd Order RC Active Filters

• Second order filters
  • 2nd order filters with gain exhibit a more complex response than 2nd order passive filters
  • Over damped: Filter response is smooth and slow
    Transient response is smooth and slow
    \[ A < 1.586 \quad Q < \frac{1}{\sqrt{2}} \quad \zeta > 0.707 \]
  • Critically damped: Filter response matches the passive filter
    \[ A = 1.586 \quad Q = \frac{1}{\sqrt{2}} \quad \zeta = 0.707 \]
  • Under damped: Filter response is peaked
    Transient response “rings”
    \[ A > 1.586 \quad Q > \frac{1}{\sqrt{2}} \quad \zeta < 0.707 \]
  • Unstable: Filter circuit oscillates independently
    \[ A \to 3 \quad Q = \infty \quad \zeta \to 0 \]
Over Damped

Critically Damped
Unstable

Under Damped
$N^{th}$ Order RC Active Filters

- Order $> 2$ active Filters
  - Cascade 1$^{st}$ and 2$^{nd}$ order stages
  - Remember, gain can be tricky in 2$^{nd}$ order stages
    - Move gain to 1$^{st}$ order stages where possible

- Computer programs and calculators available
N\textsuperscript{th} Order RC Active Filters

- Chebyshev – 5\textsuperscript{th} order HP
- High Pass

![Diagram of a Chebyshev 5\textsuperscript{th} order High Pass filter](image)

- 100dB/decade
- Linear mag plot
- Passband ripple