MODELING DC SERVOMOTORS
CONTROL SYSTEMS TECH NOTE

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INTRODUCTION

A DC motor is an actuator that converts electrical energy to mechanical rotation using the principles of electromagnetism. The circuit symbol for a DC motor is shown in Figure 1.

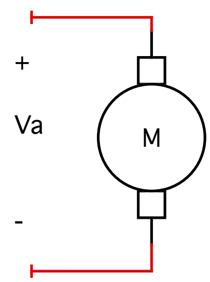

Figure 1: Circuit symbol for a DC servomotor

The learning objectives of this technical note are:

1. **Draw** the equivalent DC servomotor circuit theory model.
2. **State** the equations of motion used to derive the electromechanical transfer function in the time domain and the s-domain.
3. **Draw** the DC servomotor signal block diagram.
4. **Derive** the DC servomotor electromechanical transfer function.

CIRCUIT THEORY MODEL

The circuit shown in Figure 2 models the DC servomotor. Note that an armature control current is created when the armature control voltage, Va, energizes the motor. The current flow through a series-connected

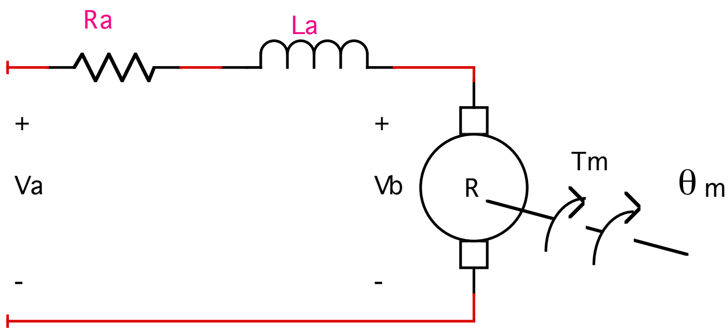

Figure 2: DC servomotor circuit theory model

armature resistance, an armature inductance, and the rotational component (the rotor) of the motor. The rotor shaft is typically drawn to the right with the torque (T_m) and angular displacement (θ_m) variables shown. The motor transfer function is the ratio of angular displacement to armature voltage.

Figure 3: The DC servomotor transfer function

EQUATIONS OF MOTION

Three equations of motion are fundamental to the derivation of the transfer function. Relationships between torque and current, voltage and angular displacement, and torque and system inertias are used.

Torque is proportional to the armature current. The constant of proportionality is called the torque constant and is given the symbol K_t. The time and frequency domain relationships are given as equations 1a and 1b.

(1a) $$T_m(t) = K_t * I_a(t)$$

(1b) $$T_m(s) = K_t * I_a(s)$$

The back electromotive force (back-emf), V_b, is a result of the rotor spinning at right angles in a magnetic field. It is proportional to the shaft velocity. The constant of proportionality is called the back-emf constant and is given the symbol K_b. The time and frequency domain relationships are given as equations 2a and 2b.

(2a) $$V_b(t) = K_b * \frac{d\theta_m}{dt}$$

(2b) $$V_b(s) = K_b * s\theta_m(s)$$

Torque can also be written as a relationship that depends on the load attached to the motor shaft. If the load inertial and damping behaviors are reflected back to the shaft the resulting equivalent inertia coefficient and viscous damping coefficient can be used to model the mechanical system attached to the shaft. These equivalents combine the motor and load properties in a model with a single degree of freedom. Figure 4

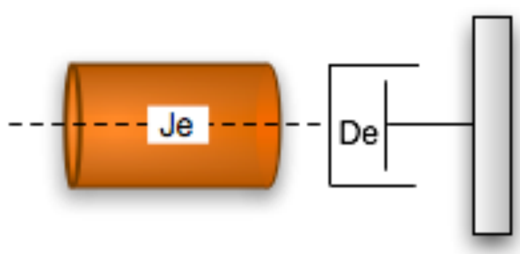

Figure 4: The mechanical system attached to the motor shaft

diagrams the equivalent system. The torque equation can be written by inspection by summing the forces acting at the shaft.

(3) $$T_m = (J_e * s^2 + D_e * s) * \theta_m$$
$$G_\theta = \frac{1}{J_e * s^2 + D_e * s}$$

DRAW THE DC SERVOMOTOR BLOCK DIAGRAM

The DC servomotor transfer function block diagram can be created through an examination of the armature current equation. Ohm's law for impedances is used to write the armature current equation in the s-domain. Note that the equation includes both real and imaginary parts of the impedance. The reactance is inductive.

(4) $$I_a = \frac{V_a - V_b}{R_a + s * L_a} = \left(\frac{1}{R_a + s * L_a}\right) * \left(V_a - V_b\right) = G_I * (V_a - V_b)$$

Figure 5 shows the block diagram model of this equation.

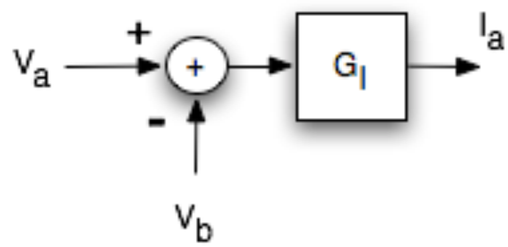

Figure 5: The block diagram after deriving armature current

Similarly, the equations of motion can be used to relate the armature current to torque, torque to angular displacement, and angular displacement to the back-emf. The complete block diagram is shown in Figure 6.

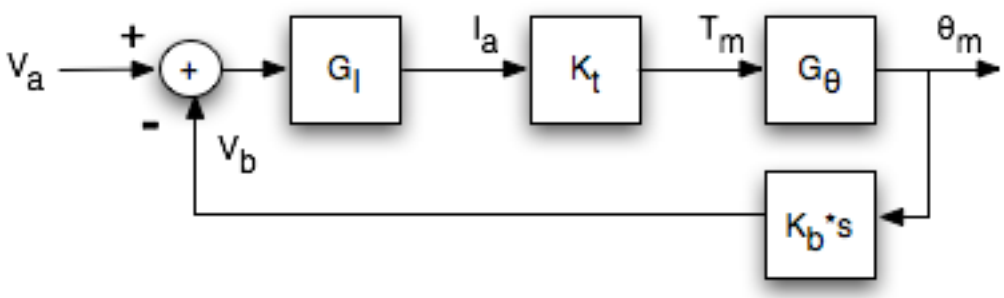

Figure 6: The complete block diagram of the DC servomotor

Note that this actuator is a feedback system that can be reduced to the standard form for derivation of the gain. The series gain path is reduced to an equivalent gain through multiplication. The reduced block diagram is shown in Figure 7. The forward path gain is traditionally called G while the feedback path gain is H.

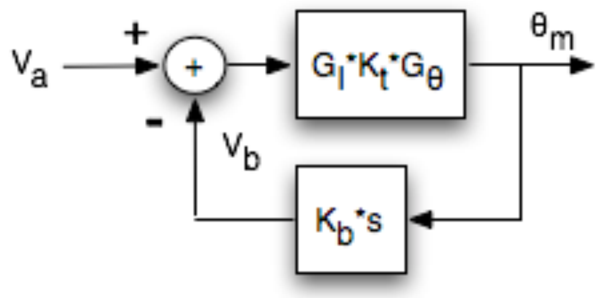

Figure 7: The reduced block diagram of the DC servomotor

DERIVE THE TRANSFER FUNCTION

The gain can be calculated directly from the reduced block diagram:

(5) $$G_m = \frac{\theta_m}{V_a} = \frac{G}{1+G*H} = \frac{G_I*K_t*G_\theta}{1+(G_I*K_t*G_\theta)*(K_b*s)}$$

Note that the denominator contains a + sign because the block diagram has negative feedback at the summation node. The denominator would contain a - sign if the block diagram used positive feedback. Remember, the denominator sign is *opposite* the feedback sign on the block diagram.

LABORATORY QUANSAR PLANT DC SERVOMOTOR

The MSOE control systems laboratory Quansar plant includes a DC servomotor with these parameters:

PARAMETER	**VALUE**
R_a	2.6 Ω
L_a	180 μH
K_t	7.67E-3 N*m/A
K_b	7.67E-3 V/(r/s)
J_e	5.3E-7 kg*m^2
D_e	7.7E-6 N*m/(r/s)

SKILL ASSESSMENT EXERCISE

Derive the motor transfer function G_m for the Quansar plant motor.