UY 6.62

Note at frequency 784 Hz.

Frequency of 9^{th} harmonic is $9(784) = 7056$ Hz

$\therefore$ Nyquist sampling rate of $2(7056) = 14112$ Hz is required

i.e., must sample at > 14112 samples/sec to prevent aliasing in reconstruction

UY 6.66

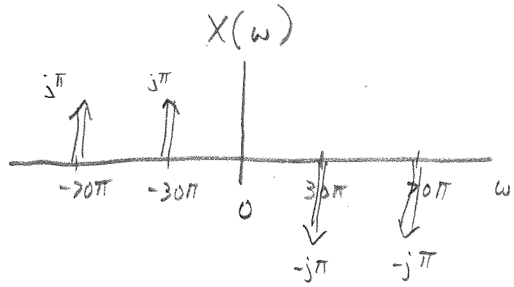
$$x(t) = \sin(30\pi t) + \sin(70\pi t)$$

sampled at 50 samples/sec

Note: From FT Table

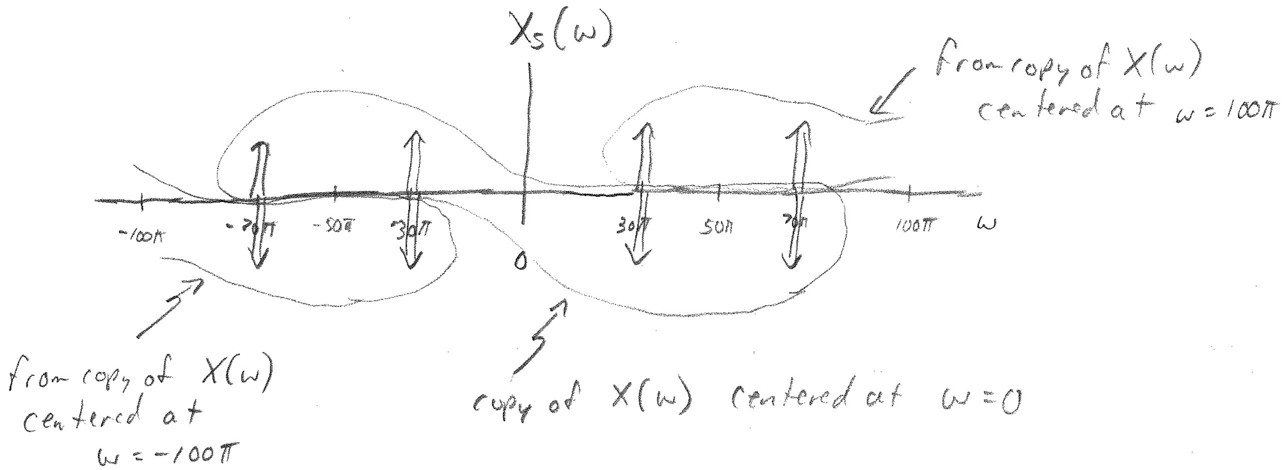
$$\sin(\omega_0 t) \xleftrightarrow{\mathcal{F}} j\pi\left[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right]$$

$$\therefore X(\omega) = j\pi\left[\delta(\omega+30\pi) - \delta(\omega-30\pi)\right] + j\pi\left[\delta(\omega+70\pi) - \delta(\omega-70\pi)\right]$$



Consider impulse-train sampling at $f_s = 50\text{ Hz}$

$$\Rightarrow \omega_s = 2\pi(50) = 100\pi \text{ rad/s}$$

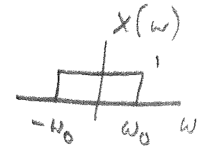


* all impulses cancel each other.

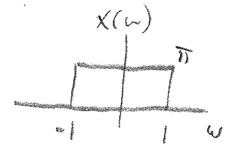
∴ output of reconstruction filter is zero.

$x(t) = \sin(t)/t$

a) From FT Table: $\dfrac{\sin(W_0 t)}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow}$

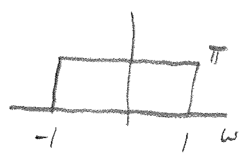


$\therefore\ x(t) = \pi\left(\dfrac{\sin(t)}{\pi t}\right) \overset{\mathcal{F}}{\longleftrightarrow}$



$$X(w) = \begin{cases} \pi, & |w| < 1 \\ 0, & \text{otherwise} \end{cases}$$

b) $|X(w)|$

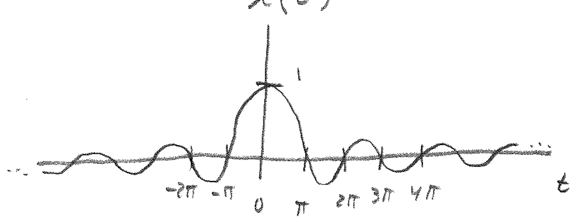


Yes. $X(w)$ is bandlimited

Max Frequency is $\underline{1}$ rad/s

c) Must choose sampling frequency > 2 rad/s

d) Consider a plot of $x(t)$:



zero-crossings occur when $\sin(t) = 0 \Rightarrow t = k\pi$

$\therefore$ choosing $T_s = k\pi$ (e.g. $T_s = \pi$) will

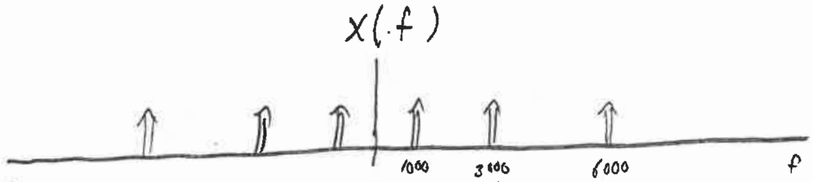
result in $x(0) = 1$ and $x(nT_s) = 0$ for $n \neq 0$

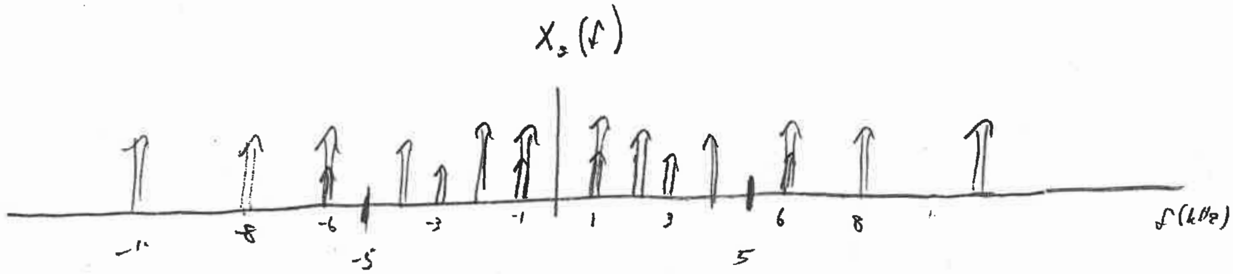
$x(t) = \cos(2000\pi t) + \sin(6000\pi t) + \cos(12000\pi t)$

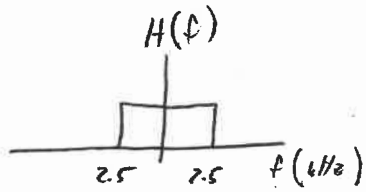
Max freq is 6000 Hz.

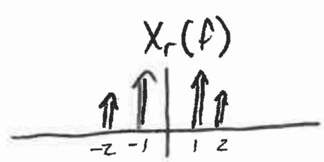
$\therefore$ Nyquist rate is 12000 samples/sec $\Rightarrow f_s > 12000$ Hz

Suppose $f_s = 5000$ Hz









* 1 kHz and 2 kHz sinusoids appear at output

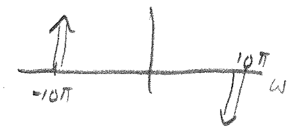
UY 6.71

a) $$x_1(t) = \frac{3t+7}{t} \sin(10\pi t)$$

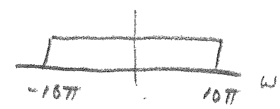
$$= 3\sin(10\pi t) + 7\,\frac{\sin(10\pi t)}{t}$$

Find FT of each component:

$$3\sin(10\pi t) \overset{\mathcal{F}}{\longleftrightarrow}$$



$$7\,\frac{\sin(10\pi t)}{t} \overset{\mathcal{F}}{\longleftrightarrow}$$



$\therefore$ Max frequency is $\omega_m = 10\pi$ rad/s

$\Rightarrow$ Nyquist sampling rate is $\omega_s = 20\pi$ rad/s

b) $$x_2(t) = \sin(6\pi t)\sin(4\pi t)$$

Using Trig Id: $\sin(\alpha)\sin(\beta) = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)$

$$x_2(t) = \tfrac{1}{2}\cos(2\pi t) - \tfrac{1}{2}\cos(10\pi t)$$

Max freq is $\omega_m = 10\pi$ rad/s

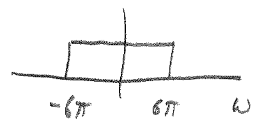
$\Rightarrow$ Nyquist sampling rate is $\omega_s = 20\pi$ rad/s

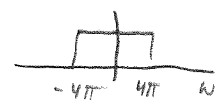
(continued)

c) $x_3(t) = \dfrac{\sin(6\pi t)\sin(4\pi t)}{t^2}$

$$= \left(\frac{\sin(6\pi t)}{t}\right)\left(\frac{\sin(4\pi t)}{t}\right)$$

Take FT of each component:

$\dfrac{\sin(6\pi t)}{t} \overset{\mathcal{F}}{\longleftrightarrow}$ 

$\dfrac{\sin(4\pi t)}{t} \overset{\mathcal{F}}{\longleftrightarrow}$ 

$X_3(\omega)$ is the <u>convolution</u> of these two rectangles

Max freq is 10π rad/s

$\therefore$ Nyquist freq is $\omega_s = 20\pi$ rad/s