UY 7.8

$$\{\underline{1}, 1\} \longrightarrow \boxed{\text{Linear}} \longrightarrow \{\underline{5}, 6\}$$

$$\delta[n] = \{\underline{1}\} \longrightarrow \boxed{\text{Linear}} \longrightarrow \{\underline{1}, 3\}$$

Define $x[n] = \{\underline{0}, 1\}$. Then, note that $\{\underline{1}, 1\} = \delta[n] + x[n]$

Since system is linear, we know that response to $\{\underline{1}, 1\}$ must be the sum of the responses to $\delta[n]$ and $x[n]$. Define $y[n]$ to be the response to $x[n]$. Then,

$$\{\underline{5}, 6\} = \{\underline{1}, 3\} + y[n]$$

$$\Rightarrow \quad y[n] = \{\underline{4}, 3\}$$

Since $x[n] = \delta[n-1]$, if the system was T.I. then $y[n]$ must be $\{\underline{0}, 1, 3\}$. This is not true, therefore the system is Time Varying.

MA system:

$$y[n] = 2x[n] - x[n-1] + 2x[n-3]$$

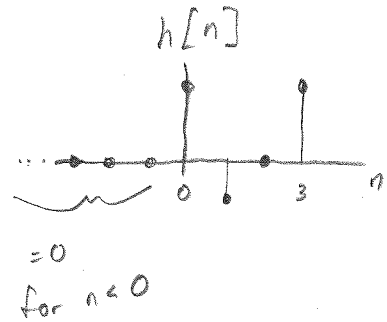
a) Let $x[n] = \delta[n]$. Then, by definition,

$y[n] = h[n]$.

$$\therefore \quad h[n] = 2\delta[n] - \delta[n-1] + 2\delta[n-3]$$

$$= \{\underline{2}, -1, 0, 2\}$$

b) system is <u>causal</u> since $h[n] = 0$ for $n < 0$



c) system is BIBO stable since $\sum_{n=-\infty}^{\infty} |h[n]| < 0$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{3} |h[n]| = 2 + 1 + 3$$

$$= 6 < \infty$$

UY 7.10

a) $\{\underline{3}, 4, 5\} * \{\underline{6}, 7, 8\}$

Let $x[n] = \{\underline{3}, 4, 5\}$ and $h[n] = \{\underline{6}, 7, 8\}$

$$\sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{k=0}^{2} x[k]\,h[n-k]$$

$$= x[0]h[n] + x[1]h[n-1] + x[2]h[n-2]$$

$$= 3\cdot\{\underline{6}, 7, 8\} + 4\{\underline{0}, 6, 7, 8\} + 5\{\underline{0}, 0, 6, 7, 8\}$$

$$= \{\underline{18}, 21, 24\} + \{\underline{0}, 24, 28, 32\} + \{\underline{0}, 0, 30, 35, 40\}$$

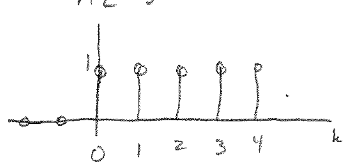
$$= \{\underline{18}, 45, 82, 67, 40\}$$

b) $\{\underline{1}, 2, -3\} * u[n]$

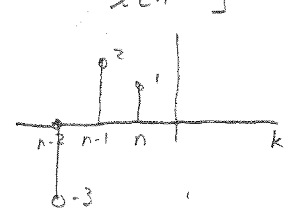
Let $x[n] = \{\underline{1}, 2, -3\}$ and $h[n] = u[n]$

$$\sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

$h[k]$



$x[n-k]$



For $n < 0$: $y[n] = 0$

For $n = 0$: $y[n] = 1\cdot 1 = 1$

For $n = 1$: $y[n] = 1\cdot 1 + 2\cdot 1 = 3$

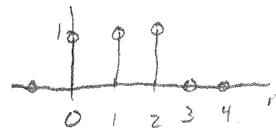
For $n = 2$: $y[n] = 1\cdot 1 + 2\cdot 1 + (-3)\cdot 1 = 0$

* For $n \geq 2$: $y[n] = 0$

$\therefore$ Result is

$\{\underline{1}, 3\}$

c) $\{\underline{3}, 4, 5\} * (u[n] - u[n-3])$



Let $x[n] = \{\underline{3}, 4, 5\}$ and $h[n] = \{\underline{1}, 1, 1\}$

$$\sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \sum_{k=0}^{2} x[k]\, h[n-k]$$

$$= x[0]\, h[n] + x[1]\, h[n-1] + x[2]\, h[n-2]$$

$$= 3 \cdot \{\underline{1}, 1, 1\} + 4\{\underline{0}, 1, 1, 1\} + 5\{\underline{0}, 0, 1, 1, 1\}$$

$$= \{\underline{3}, 3, 3\} + \{\underline{0}, 4, 4, 4\} + \{\underline{0}, 0, 5, 5, 5\}$$

$$= \{\underline{3}, 7, 12, 9, 5\}$$

d) $\{\underline{1}, 2, 4\} * 2\delta[n-2]$

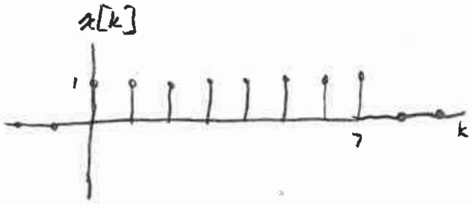
Note: $x[n] * \alpha\delta[n-M] = \alpha\, x[n-M]$

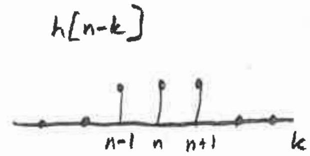
$$\therefore \{\underline{1}, 2, 4\} * 2\delta[n-2] = \{\underline{0}, 0, 2, 4, 8\}$$

$x[n] = u[n] - u[n-8]$

$h[n] = u[n+1] - u[n-2]$

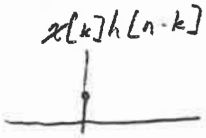
Solve $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$





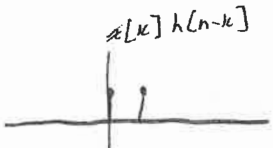
for $n+1 < 0$ or $n < -1$: No overlap $\Rightarrow y[n] = 0$

for $n+1 = 0$ or $n = -1$:



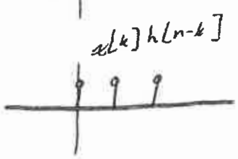
$\Rightarrow y[-1] = 1$

for $n+1 = 1$ or $n = 0$:



$\Rightarrow y[0] = 2$

for $n+1 = 2$ or $n = 1$



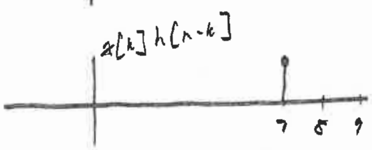
$\Rightarrow y[1] = 3$

(this result continues until)

for $n+1 = 8$ or $n = 7$:

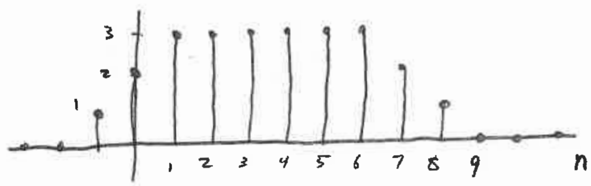
$\Rightarrow y[7] = 2$

for $n+1 = 9$ or $n = 8$:



$\Rightarrow y[8] = 1$

for $n-1 > 7$: $n > 8$ No overlap $\Rightarrow y[n] = 0$



UY 7.13

$$x[n] \longrightarrow \boxed{h_1[n]} \longrightarrow w[n] = 3x[n] - 2x[n-1]$$

and

$$w[n] \longrightarrow \boxed{h_2[n]} \longrightarrow y[n] = 5w[n] - 4w[n-1]$$

Since systems are connected in series, the overall impulse response is $h_1[n] * h_2[n]$

Find $h_1[n]$ by letting $x[n] = \delta[n]$ (and therefore $w[n] = h_1[n]$)

$$\Rightarrow h_1[n] = 3\delta[n] - 2\delta[n-1]$$
$$= \{\underline{3}, -2\}$$

Find $h_2[n]$ by letting $w[n] = \delta[n]$ (and therefore $y[n] = h_2[n]$)

$$\Rightarrow h_2[n] = 5\delta[n] - 4\delta[n-1]$$
$$= \{\underline{5}, -4\}$$

$$h_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} h_1[k]\, h_2[n-k]$$
$$= 3 \cdot \{\underline{5}, -4\} + (-2) \cdot \{\underline{0}, 5, -4\}$$
$$= \{\underline{15}, -12\} + \{\underline{0}, -10, +8\}$$
$$= \{\underline{15}, -22, 8\}$$