UY 7-15 (a), (b), (c)

a) $(1+2^n)u[n]$

$= u[n] + 2^n u[n]$

$u[n] \overset{z}{\longleftrightarrow} \frac{z}{z-1} \qquad |z| > 1$

$2^n u[n] \overset{z}{\longleftrightarrow} \frac{z}{z-2} \qquad |z| > 2$

$\therefore \frac{z}{z-1} + \frac{z}{z-2} = \frac{z(z-2) + z(z-1)}{(z-1)(z-2)}$

$= \frac{2z^2 - 3z}{z^2 - 3z + 2} \qquad \text{ROC: } |z| > 2$

b) $2^n u[n] + 3^n u[n]$

z-Transform Pair: $a^n u[n] \overset{z}{\longleftrightarrow} \frac{z}{z-a} \qquad |z| > |a|$

$\therefore \frac{z}{z-2} + \frac{z}{z-3} = \frac{z(z-3) + z(z-2)}{(z-2)(z-3)}$

$= \frac{2z^2 - 5z}{z^2 - 5z + 6} \qquad \text{ROC: } |z| > 3$

c) $\{\underline{1}, -2\} + 2^n u[n]$

ROC: all z

ROC: $z \neq 0$

$$\{\underline{1}, -2\} = \delta[n] - 2\delta[n-1] \xleftrightarrow{z} 1 - 2z^{-1}$$

$$2^n u[n] \xleftrightarrow{z} \frac{z}{z-2} \qquad \text{ROC: } |z| > 2$$

$$\therefore \quad 1 - 2z^{-1} + \frac{z}{z-2} = 1 - \frac{2}{z} + \frac{z}{z-2}$$

$$= \frac{z-2}{z} + \frac{z}{z-2}$$

$$= \frac{(z-2)(z-2)}{z(z-2)} + \frac{z^2}{z(z-2)}$$

$$= \frac{z^2 - 4z + 4 + z^2}{z^2 - 2z}$$

$$= \frac{2z^2 - 4z + 4}{z^2 - 2z} \qquad \text{ROC: } |z| > 2$$

UY 7-17 a, b, c, d

a) $$\frac{z+1}{2z} = \frac{1}{2} + \frac{1}{2z}$$

$$= \frac{1}{2} + \frac{1}{2} z^{-1}$$

$$\mathcal{Z}^{-1}\left\{ \frac{1}{2} + \frac{1}{2} z^{-1} \right\} = \left\{ \underline{\tfrac{1}{2}}, \tfrac{1}{2} \right\} = \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-1]$$

b) $$\frac{z-1}{z-2} = \frac{z}{z-2} - \frac{1}{z-2}$$

$$\mathcal{Z}^{-1}\left\{ \frac{z}{z-2} \right\} = 2^n u[n]$$

$$\mathcal{Z}^{-1}\left\{ \frac{1}{z-2} \right\} = 2^{n-1} u[n-1]$$

$$\therefore \quad 2^n u[n] - 2^{n-1} u[n-1]$$

c) $$\frac{2z+3}{z^2(z+1)} = z^{-2}\left(\frac{2z+3}{z+1} \right)$$

$$= z^{-2}\left(\frac{2z}{z+1} + \frac{3}{z+1} \right)$$

$$\mathcal{Z}^{-1}\left\{ \frac{2z}{z-(-1)} \right\} = 2(-1)^n u[n]$$

$$\mathcal{Z}^{-1}\left\{ \frac{3}{z-(-1)} \right\} = 3(-1)^{n-1} u[n-1]$$

Right sample shift by 2

$$\therefore \quad 2(-1)^{n-2} u[n-2] + 3(-1)^{n-3} u[n-3]$$

Alternate solution for c) using partial fractions:

$$\frac{2z+3}{z^2(z+1)} = \frac{A}{z^2} + \frac{B}{z} + \frac{C}{z+1}$$

$$2z+3 = A(z+1) + Bz(z+1) + Cz^2$$

$z=0$: $\quad 3 = A \quad \Rightarrow \quad \boxed{A=3}$

$z=-1$: $\quad 1 = C(-1)^2 \quad \Rightarrow \quad \boxed{C=1}$

$z=1$: $\quad 5 = 2A + 2B + C$

$\quad = 6 + 2B + 1 \quad \Rightarrow \quad \boxed{B=-1}$

$$\therefore \frac{2z+3}{z^2(z+1)} = \frac{3}{z^2} + \frac{-1}{z} + \frac{1}{z+1}$$

$$\mathcal{Z}^{-1}\left\{\frac{3}{z^2}\right\} = 3\delta[n-2]$$

$$\mathcal{Z}^{-1}\left\{-\frac{1}{z}\right\} = -\delta[n-1]$$

$$\mathcal{Z}^{-1}\left\{\frac{1}{z+1}\right\} = (-1)^{n-1}u[n-1]$$

$$\therefore \quad 3\delta[n-2] - \delta[n-1] + (-1)^{n-1}u[n-1]$$

d) $$\frac{z^2+3z}{z^2+3z+2} = \frac{z(z+3)}{z^2+3z+2} = \frac{z(z+3)}{(z+1)(z+2)}$$

Since numerator and denominator have same degree, first do long division

$$\begin{array}{r|l} & \quad 1 \\ z^2+3z+2 & z^2+3z \\ & -\; z^2+3z+2 \\ \hline & \qquad -2 \end{array}$$

$$\therefore \frac{z^2+3z}{z^2+3z+2} = 1 - \frac{2}{z^2+3z+2}$$

$$= 1 - \underbrace{\frac{2}{(z+1)(z+2)}}_{\text{Do Partial Fractions}}$$

$$\frac{2}{(z+1)(z+2)} = \frac{A}{(z+1)} + \frac{B}{(z+2)}$$

$$\Rightarrow \quad 2 = A(z+2) + B(z+1)$$

$z=-1$: $\quad 2 = A(-1+2) \Rightarrow A = 2$

$z=-2$: $\quad 2 = B(-2+1) \Rightarrow B = -2$

$$\therefore \quad 1 - \left(\frac{2}{z+1} - \frac{2}{z+2}\right)$$

Take inverse Z-Transform of each:

$$\delta[n] - 2(-1)^{n-1}u[n-1] + 2(-2)^{n-1}u[n-1]$$

Example:

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$x[n] = 2\delta[n] - 3\delta[n-2] + 4\delta[n-3] \qquad = \{\underline{2}, -3, 4\}$$

$$h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2] \qquad = \{\underline{1}, 2, 1\}$$

Find $y[n]$

$$X(z) = 2 - 3z^{-2} + 4z^{-3}$$

$$H(z) = 1 + 2z^{-1} + z^{-2}$$

$$Y(z) = H(z)X(z)$$

$$= (1 + 2z^{-1} + z^{-2})(2 - 3z^{-2} + 4z^{-3})$$

$$= 2 - 3z^{-2} + 4z^{-3} + 4z^{-1} - 6z^{-3} + 8z^{-4}$$
$$+ 2z^{-2} - 3z^{-4} + 4z^{-5}$$

$$Y(z) = 2 + 4z^{-1} - z^{-2} - 2z^{-3} + 5z^{-4} + 4z^{-5}$$

$$y[n] = \{\underline{2}, 4, -1, -2, 5, 4\}$$

★ polynomial multiplication in Z-domain is equivalent to convolution in time-domain

$$x[n] = \{\underline{1}, 1\}$$

$$= \delta[n] + \delta[n-1]$$

$$\therefore X(z) = 1 + z^{-1}$$

$$y[n] = \{\underline{1}, 2, 2, 1\}$$

$$\therefore Y(z) = 1 + 2z^{-1} + 2z^{-2} + z^{-3}$$

$$H(z) = Y(z)/X(z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 + z^{-1}}$$

$$= \frac{z^3 + 2z^2 + 2z + 1}{z^2(z+1)}$$

$$= \frac{(z+1)(z^2 + z + 1)}{z^2(z+1)}$$

$$= \frac{z^2 + z + 1}{z^2}$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2}$$

$$\therefore \quad h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$$

$$= \{\underline{1}, 1, 1\}$$