UY 7-23

$$u[n] \longrightarrow \boxed{LTI} \longrightarrow 2u[n] + (-2)^n u[n]$$

a) $x[n] = u[n] \Rightarrow X(z) = \frac{z}{z-1}$

$y[n] = 2u[n] + (-2)^n u[n] \Rightarrow Y(z) = 2\frac{z}{z-1} + \frac{z}{z+2}$

$$H(z) = Y(z)/X(z) = \frac{\left(\frac{2z}{z-1} + \frac{z}{z+2}\right)}{\left(\frac{z}{z-1}\right)} = 2 + \frac{z-1}{z+2}$$

$$= \frac{2z+4}{z+2} + \frac{z-1}{z+2}$$

$$= \frac{3z+3}{z+2}$$

b) $H(z) = \frac{3(z+1)}{z+2} \Rightarrow$ zero at $z = -1$

pole at $z = -2$

c) $H(z) = 3\frac{z}{z+2} + 3\frac{1}{z+2}$

$$\therefore h[n] = 3(-2)^n u[n] + 3(-2)^{n-1} u[n-1]$$

d) $H(z) = \frac{3z+3}{z+2} = \frac{Y(z)}{X(z)}$

$$Y(z)[z+2] = X(z)[3z+3]$$

Multiply both sides by z^{-1}

$$Y(z)[1+2z^{-1}] = X(z)[3+3z^{-1}]$$

$$\therefore y[n] + 2y[n-1] = 3x[n] + 3x[n-1]$$

UY 7.25

$$H(z) = \frac{(z-1)(z-6)}{(z-2)(z-3)}$$

a) zeros: $z=1$ and $z=6$

poles: $z=2$ and $z=3$

* System is **NOT** BIBO stable since poles are outside unit circle

b) $$H(z) = \frac{z^2 - 7z + 6}{z^2 - 5z + 6}$$

$$= \frac{1 - 7z^{-1} + 6z^{-2}}{1 - 5z^{-1} + 6z^{-2}} = Y(z)/X(z)$$

$$\therefore Y(z)\left[1 - 5z^{-1} + 6z^{-2}\right] = X(z)\left[1 - 7z^{-1} + 6z^{-2}\right]$$

$$\therefore y[n] - 5y[n-1] + 6y[n-2] = x[n] - 7x[n-1] + 6x[n-2]$$

c) $x[n] = \{\underline{1}, -5, 6\} \Rightarrow X(z) = 1 - 5z^{-1} + 6z^{-2}$

$$Y(z) = H(z)X(z) = \left(\frac{1 - 7z^{-1} + 6z^{-2}}{1 - 5z^{-1} + 6z^{-2}}\right)\left(1 - 5z^{-1} + 6z^{-2}\right)$$

$$= 1 - 7z^{-1} + 6z^{-2}$$

$$\therefore y[n] = \{\underline{1}, -7, 6\}$$

(continued)

d) $$H(z) = \frac{(z-1)(z-6)}{(z-2)(z-3)}$$

$$H(z)\left(\frac{1}{z}\right) = \frac{(z-1)(z-6)}{z(z-2)(z-3)} = \frac{A}{z} + \frac{B}{z-2} + \frac{C}{z-3}$$

$$(z-1)(z-6) = A(z-2)(z-3) + Bz(z-3) + Cz(z-2)$$

$z=0$: $(-1)(-6) = A(-2)(-3) \Rightarrow A = 1$

$z=2$: $(1)(-4) = B(2)(-1) \Rightarrow B = 2$

$z=3$: $(2)(-3) = C(3)(1) \Rightarrow C = -2$

$$\therefore H(z)\left(\frac{1}{z}\right) = \frac{1}{z} + \frac{2}{z-2} - \frac{2}{z-3}$$

$$H(z) = 1 + 2\frac{z}{z-2} - 2\frac{z}{z-3}$$

$$\therefore h[n] = \delta[n] + 2(2)^n u[n] - 2(3)^n u[n]$$

UY 7.29

a) $y[n] = x[n] + 0.5\,x[n-1] + x[n-2]$

$Y(z) = X(z) + 0.5\,z^{-1}X(z) + z^{-2}X(z)$

$\therefore H(z) = Y(z)/X(z) = 1 + 0.5\,z^{-1} + z^{-2}$

$\therefore H(e^{j\Omega}) = H(z)\Big|_{z=e^{j\Omega}}$

$= 1 + 0.5\,e^{-j\Omega} + e^{-j2\Omega}$

b) $\cos\left(\frac{\pi}{2}n\right)$

$\Omega_0 = \pi/2 \Rightarrow N_0 = \frac{2\pi}{\Omega_0} = 4$

Expand using FS: $\cos\left(\frac{\pi}{2}n\right) = \frac{1}{2}e^{j\pi/2\,n} + \frac{1}{2}e^{-j\pi/2\,n}$

$= \frac{1}{2}e^{j\pi/2\,n} + \frac{1}{2}e^{j3\pi/2\,n}$

$= \sum_{n=0}^{3} X_k\, e^{jk\frac{\pi}{2}n}$

$\therefore X_0 = 0$

$X_1 = \frac{1}{2}$

$X_2 = 0$

$X_3 = \frac{1}{2}$

(continued)

$$Y_k = H\left(e^{jk\Omega_0}\right) X_k$$

$$= H\left(e^{jk\pi/2}\right) X_k$$

$$H\left(e^{j\pi/2}\right) = 1 + 0.5e^{-j\pi/2} + e^{-j\pi}$$

$$= 1 - 0.5j - 1$$

$$= -0.5j \qquad \Rightarrow \qquad Y_1 = -\tfrac{1}{4}j$$

$$H\left(e^{j3\pi/2}\right) = 1 + 0.5e^{-j3\pi/2} + e^{-j3\pi}$$

$$= 1 + 0.5j - 1$$

$$= 0.5j \qquad \Rightarrow \qquad Y_3 = \tfrac{1}{4}j$$

$$\therefore \quad y[n] = \sum_{k=0}^{3} Y_k e^{jk\frac{\pi}{2}n}$$

$$= Y_1 e^{j\pi/2\, n} + Y_3 e^{j3\pi/2\, n}$$

$$= -\tfrac{1}{4}j\, e^{j\frac{\pi}{2}n} + \tfrac{1}{4}j\, e^{j\frac{3\pi}{2}n}$$

$$= -\tfrac{1}{4}j\, e^{j\frac{\pi}{2}n} + \tfrac{1}{4}j\, e^{-j\frac{\pi}{2}n}$$

$$\sin\theta = \frac{j}{2}\left[e^{-j\theta} - e^{j\theta}\right]$$

$$= \frac{1}{2}\sin\left(\frac{\pi}{2}n\right)$$

UY 7.33

a) $y[n] + y[n-1] = x[n] - x[n-1]$

$$Y(z) + z^{-1}Y(z) = X(z) - z^{-1}X(z)$$

$$Y(z)\left[1+z^{-1}\right] = X(z)\left[1-z^{-1}\right]$$

$$\therefore H(z) = \frac{1-z^{-1}}{1+z^{-1}}$$

$$\therefore H(e^{j\Omega}) = \frac{1-e^{-j\Omega}}{1+e^{-j\Omega}} = \frac{e^{j\Omega}-1}{e^{j\Omega}+1}$$

b) input is $x[n] = 3 + 4\cos\left(\frac{\pi}{2}n + \pi/4\right)$

$\Omega_0 = \pi/2 \Rightarrow N_0 = 4.$

Expand $x[n]$ as a DTFS:

$$x[n] = \sum_{k=0}^{3} X_k\, e^{jk\frac{\pi}{2}n}$$

$$= 3 + 2e^{j\left(\frac{\pi}{2}n + \pi/4\right)} + 2e^{-j\left(\frac{\pi}{2}n + \pi/4\right)}$$

$$= \underset{X_0 = 3}{3} + \underset{X_1 = 2e^{j\pi/4}}{2e^{j\frac{\pi}{4}}}\, e^{j\frac{\pi}{2}n} + 2e^{-j\pi/4}\, e^{-j\pi/2\, n}$$

since $e^{-j\pi/2\, n} = e^{j\frac{3\pi}{2}n}$

$X_3 = 2e^{-j\pi/4}$

(continued)

Need $H(e^{j\Omega})$ evaluated at $\Omega = 0, \pi/2, 3\pi/2$

$$H(e^{j0}) = \frac{1-e^{-j0}}{1+e^{-j0}} = 0$$

$$H(e^{j\pi/2}) = \frac{1-e^{-j\pi/2}}{1+e^{-j\pi/2}} = \frac{1-(-j)}{1+(-j)} = \frac{1+j}{1-j} = j$$

$$= 1e^{j\pi/2}$$

$$H(e^{j3\pi/2}) = \frac{1-e^{-j3\pi/2}}{1+e^{-j3\pi/2}} = \frac{1-(j)}{1+(j)} = \frac{1-j}{1+j} = -j$$

$$= 1e^{-j\pi/2}$$

$$\therefore Y_0 = 0 \quad , \quad Y_1 = 2e^{j\pi/4} \cdot 1e^{j\pi/2} = 2e^{j3\pi/4}$$

$$Y_3 = 2e^{-j\pi/4} \cdot 1e^{-j\pi/2} = 2e^{-j3\pi/4}$$

$$\therefore \quad y[n] = 2e^{j3\pi/4} e^{j\pi/2 n} + 2e^{-j3\pi/4} e^{-j\frac{\pi}{2}n}$$

$$= 2\left(e^{j(\pi/2 n + 3\pi/4)} + e^{-j(\frac{\pi}{2}n + 3\pi/4)}\right)$$

$$= 4\cos\left(\frac{\pi}{2}n + 3\pi/4\right)$$

UY 7.36

$$x[n] = \cos\left(2\frac{\pi}{3}n\right) + \cos\left(\frac{\pi}{2}n\right)$$

$$y[n] + a\,y[n-1] = x[n] + b\,x[n-1] + x[n-2]$$

Find a, b such that $y[n] = \sin\left(\frac{\pi}{2}n\right)$

$*$ We need $H\left(e^{j\frac{2\pi}{3}}\right) = 0$ AND $\underbrace{H\left(e^{j\frac{\pi}{2}}\right) = e^{-j\pi/2} = -j}$

this will give $-\pi/2$ phase shift needed to turn "$\cos\left(\frac{\pi}{2}n\right)$" into "$\sin\left(\frac{\pi}{2}n\right)$"

Find $H(e^{j\Omega})$:

$$Y(z) + a z^{-1} Y(z) = X(z) + b z^{-1} X(z) + z^{-2} X(z)$$

$$H(z) = Y(z)/X(z) = \frac{1 + b z^{-1} + z^{-2}}{1 + a z^{-1}}$$

$$\therefore H(e^{j\Omega}) = \frac{1 + b e^{-j\Omega} + e^{-j2\Omega}}{1 + a e^{-j\Omega}}$$

We need: $$\frac{1 + b e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}}}{1 + a e^{-j2\pi/3}} = 0 \qquad (*)$$

$$\frac{1 + b e^{-j\pi/2} + e^{-j\pi}}{1 + a e^{-j\pi/2}} = \frac{-jb}{1 - ja} = -j \qquad (**)$$

(continued)

In the previous equation, (#) if a and b are real-valued, then choose

$$b = 1, \ a = 0$$

* Check to make sure these values work in (+)

$$\frac{1 + e^{-j2\pi/3} + e^{-j4\pi/3}}{1} = 0 \ !$$

$\therefore$ The system must be

$$y[n] = x[n] + x[n-1] + x[n-2]$$