$$x[n] = u[n+2] - u[n-3]$$

$$= \{1, 1, \underline{1}, 1, 1\}$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$

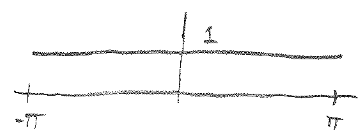
$$= \sum_{n=-2}^{2} (1)\, e^{-j\Omega n}$$

$$= e^{j2\Omega} + e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega}$$

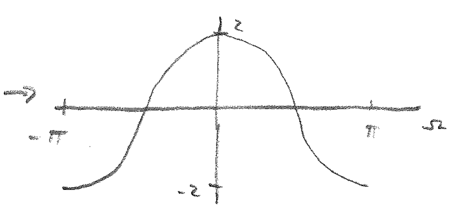
$$\cos\theta = \tfrac{1}{2} e^{j\theta} + \tfrac{1}{2} e^{-j\theta}$$

$$\therefore X(e^{j\Omega}) = 1 + 2\cos(\Omega) + 2\cos(2\Omega)$$

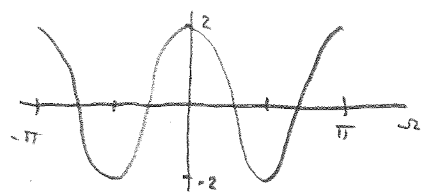
Plot: $1 \rightarrow$



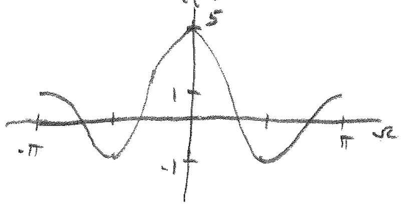
$2\cos(\Omega) \rightarrow$



$2\cos(2\Omega) \rightarrow$

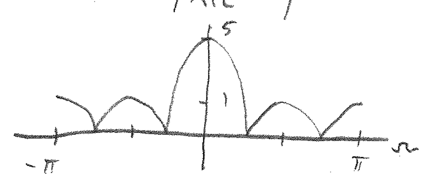


$X(e^{j\Omega})$



$\Rightarrow$

$|X(e^{j\Omega})|$



Ω	$X(e^{j\Omega})$
0	5
$\pm\pi$	1
$\pm\pi/2$	-1

$h[n] = \{ \underline{\tfrac{1}{3}}, \tfrac{1}{3}, \tfrac{1}{3} \}$

$$H(e^{j\Omega}) = \frac{1}{3}\left[1 + e^{-j\Omega} + e^{-j2\Omega}\right]$$

$$= \frac{1}{3} e^{-j\Omega}\left[e^{j\Omega} + 1 + e^{-j\Omega}\right]$$

$$= \frac{1}{3} e^{-j\Omega}\left(1 + 2\cos(\Omega)\right)$$

$$|H(e^{j\Omega})| = \left|\frac{1}{3} e^{-j\Omega}\left(1 + 2\cos(\Omega)\right)\right|$$

$$= \frac{1}{3}\left|e^{-j\Omega}\right|\left|1 + 2\cos(\Omega)\right|$$

$$= \frac{1}{3}\left|1 + 2\cos(\Omega)\right|$$

$$\measuredangle H(e^{j\Omega}) = \measuredangle\left(\frac{1}{3} e^{-j\Omega}\left(1 + 2\cos(\Omega)\right)\right)$$

$$= \left(\measuredangle \frac{1}{3} e^{-j\Omega}\right) + \measuredangle\left(1 + 2\cos(\Omega)\right)$$

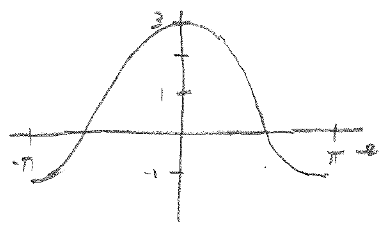
$$= -\Omega + \measuredangle\left(1 + 2\cos(\Omega)\right)$$

$= 0$ when term is positive

$= \pm\pi$ when term is negative

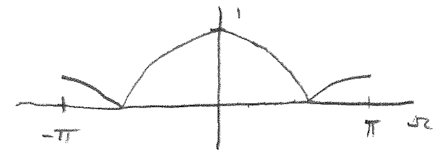
(continued)

$1 + 2\cos(\Omega)$

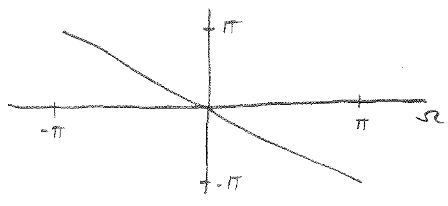


$\Rightarrow$

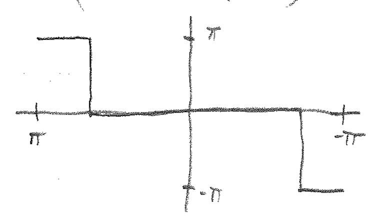
$|H(e^{j\Omega})| = \frac{1}{3}\left|1 + 2\cos(\Omega)\right|$



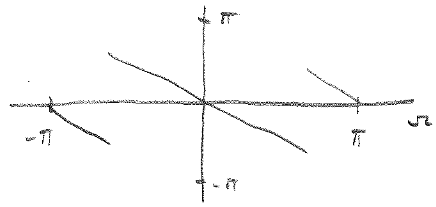
$\measuredangle e^{-j\Omega}$



$\measuredangle\left(1 + 2\cos(\Omega)\right)$



$\measuredangle H(e^{j\Omega})$



UY 7.46

$$X(e^{j\Omega}) = 3 + 2\cos(\Omega) + 4\cos(2\Omega) + j\left[6\sin(\Omega) + 8\sin(2\Omega)\right]$$

*write each cos/sin term as complex exponentials

$$\cos\theta = \tfrac{1}{2}e^{j\theta} + \tfrac{1}{2}e^{-j\theta} \Rightarrow 2\cos\theta = e^{j\theta} + e^{-j\theta}$$

$$\sin\theta = \tfrac{1}{2j}e^{j\theta} - \tfrac{1}{2j}e^{-j\theta} \Rightarrow 2j\sin\theta = e^{j\theta} - e^{-j\theta}$$

$$\therefore X(e^{j\Omega}) = 3 + e^{j\Omega} + e^{-j\Omega} + 2e^{j2\Omega} + 2e^{-j2\Omega}$$

$$+ 3e^{j\Omega} - 3e^{-j\Omega} + 4e^{j2\Omega} - 4e^{-j2\Omega}$$

$$= 3 + 4e^{j\Omega} - 2e^{-j\Omega} + 6e^{j2\Omega} - 2e^{-j2\Omega}$$

The DTFT is, by definition

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

$\therefore$ we can simply read off the $x[n]$ values

$x[0] = 3$

$x[1] = -2$

$x[-1] = 4$

$x[2] = -2$

$x[-2] = 6$

$$\therefore x[n] = \{6, 4, \underline{3}, -2, -2\}$$

UY 7.48 (b)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j3\Omega}\, 2\cos(3\Omega)\, d\Omega$$

Compare to inverse DTFT:

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})\, e^{j\Omega n}\, d\Omega$$

* Integral above is $x[3]$ for $X(e^{j\Omega}) = 2\cos(3\Omega)$

i.e., $$x[n]\Big|_{n=3} = \left[\frac{1}{2\pi}\int_{-\pi}^{\pi} 2\cos(3\Omega)\, e^{j\Omega n}\right]\Bigg|_{n=3}$$

$$2\cos(3\Omega) = 2\left[\frac{1}{2}e^{j3\Omega} + \frac{1}{2}e^{-j3\Omega}\right]$$

$$= e^{j3\Omega} + e^{-j3\Omega}$$

Since $\delta[n-m] \xleftrightarrow{\mathcal{F}} e^{-jm\Omega}$

$$x[n] = \delta[n+3] + \delta[n-3]$$

$$\therefore\ x[3] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j3\Omega}\, 2\cos(3\Omega)\, d\Omega$$

$$= \underline{1}\ //$$