$x[n] = \delta[n]$

10 point DFT: $\displaystyle X[k] = \sum_{n=0}^{9} x[n]\, e^{-j\frac{2\pi}{N}kn}$ for $k=0,1,\dots,9$

$$= \sum_{n=0}^{9} \delta[n]\, e^{-j\frac{2\pi}{N}kn}$$

* the only non-zero term is $n=0$ since

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & \text{otherwise} \end{cases}$$

$$X[k] = \delta[0]\, e^{-j\frac{2\pi}{N}k(0)}$$

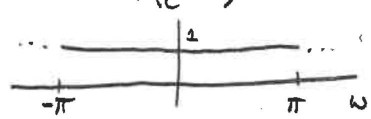
$$= 1$$

$\therefore X[k] = 1$ for $k=0,1,\dots,9$

Consider DTFT of $\delta[n]$:

$$\delta[n] \xleftrightarrow{\mathcal{F}} 1$$

$X(e^{j\omega})$



Notice that $X[k]$ consists of samples of $X(e^{j\omega})$, which are always 1.

$$x[n] = \{3, 4, 1, 2\}$$

$$X_4[k] = \sum_{n=0}^{3} x[n]\, W_4^{kn} \qquad \text{where } W_4 = e^{-j\frac{2\pi}{4}} = -j$$

$$= \sum_{n=0}^{3} x[n]\, (-j)^{nk}$$

$$X[0] = \sum_{n=0}^{3} x[n]\, (-j)^0$$

$$= \sum_{n=0}^{3} x[n] = 3+4+1+2 = 10$$

$$X[1] = \sum_{n=0}^{3} x[n]\, (-j)^n$$

$$= 3(-j)^0 + 4(-j)^1 + 1(-j)^2 + 2(-j)^3$$

$$= 3(1) + 4(-j) + 1(-1) + 2(j)$$

$$= 2 - 2j$$

$$X[2] = \sum_{n=0}^{3} x[n]\, (-j)^{2n}$$

$$= 3(-j)^0 + 4(-j)^2 + 1(-j)^4 + 2(-j)^6$$

$$= 3(1) + 4(-1) + 1(1) + 2(-1)$$

$$= -2$$

$$X[3] = \sum_{n=0}^{3} x[n]\, (-j)^{3n}$$

$$= 3(-j)^0 + 4(-j)^3 + 1(-j)^6 + 2(-j)^9$$

$$= 3(1) + 4(j) + 1(-1) + 2(-j)$$

$$= 2 + 2j$$

$$X[k] = \{10,\ 2-2j,\ -2,\ 2+2j\}$$

Using radix-2 FFT: $x[n] = \{3, 4, 1, 2\}$

Step 1:

$$g_1 = x[0] + x[2] = 4$$

$$g_2 = x[1] + x[3] = 6$$

$$h_1 = x[0] - x[2] = 2$$

$$h_2 = x[1] - x[3] = 2$$

Step 2:

$$X[0] = g_1 + g_2 = 10$$

$$X[1] = h_1 - jh_2 = 2 - j2$$

$$X[2] = g_1 - g_2 = -2$$

$$X[3] = h_1 + jh_2 = 2 + j2$$

$$X[k] = \{10, 2-j2, -2, 2+j2\}$$

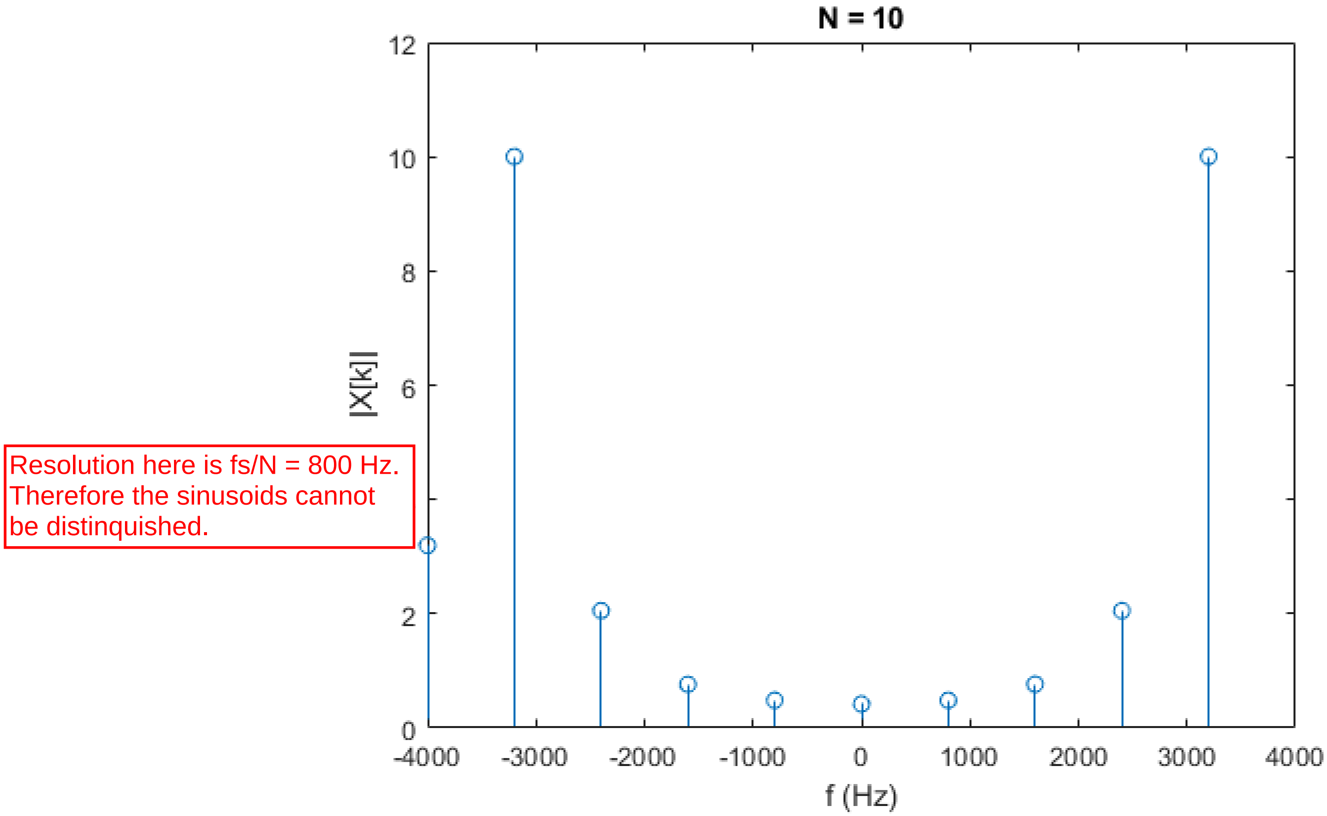
Table of Contents


```
clear all
close all

fs = 8000;
Ts = 1/fs;
```

N = 10 case

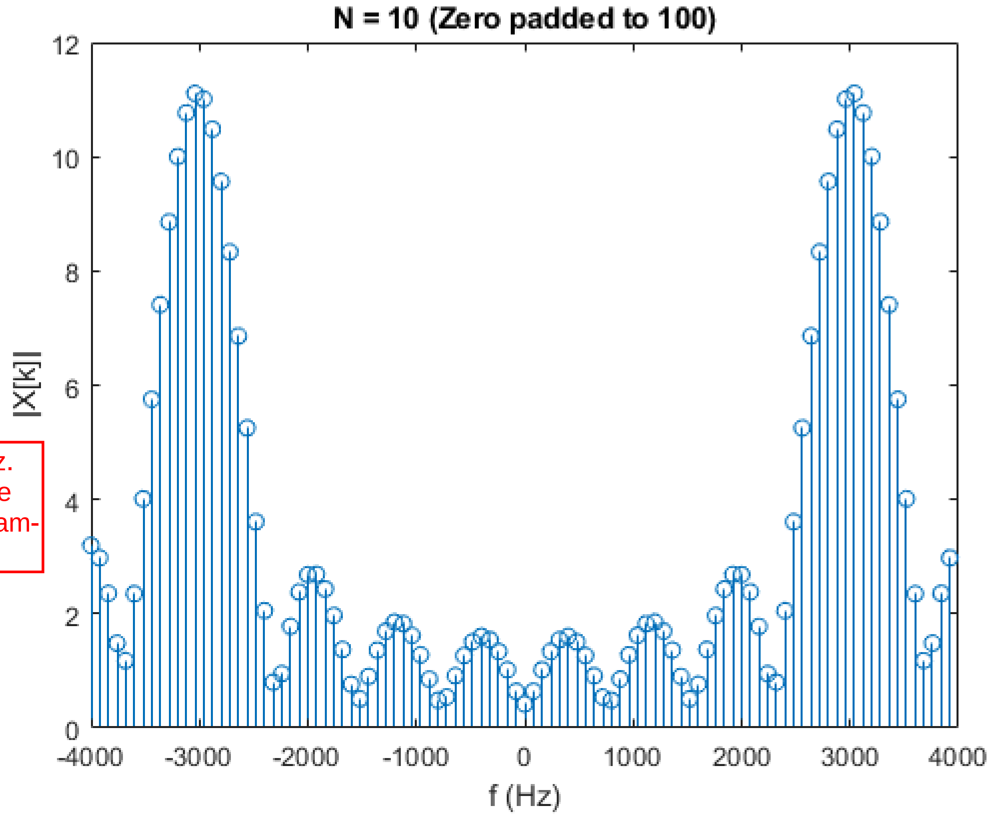
```
N = 10;
n = 0:(N-1);
x = cos(2*pi*3000*(n*Ts))+cos(2*pi*3050*(n*Ts));
k = 0:(N-1);
Omega = (-N/2:(N/2-1))*2*pi/N;
f = Omega*fs/(2*pi);
X = fftshift(fft(x));
figure
stem(f,abs(X))
title('N = 10')
xlabel('f (Hz)')
ylabel('|X[k]|')
```



Zero pad N = 10 case to 100 point DFT

Note: Do not recompute x[n]!

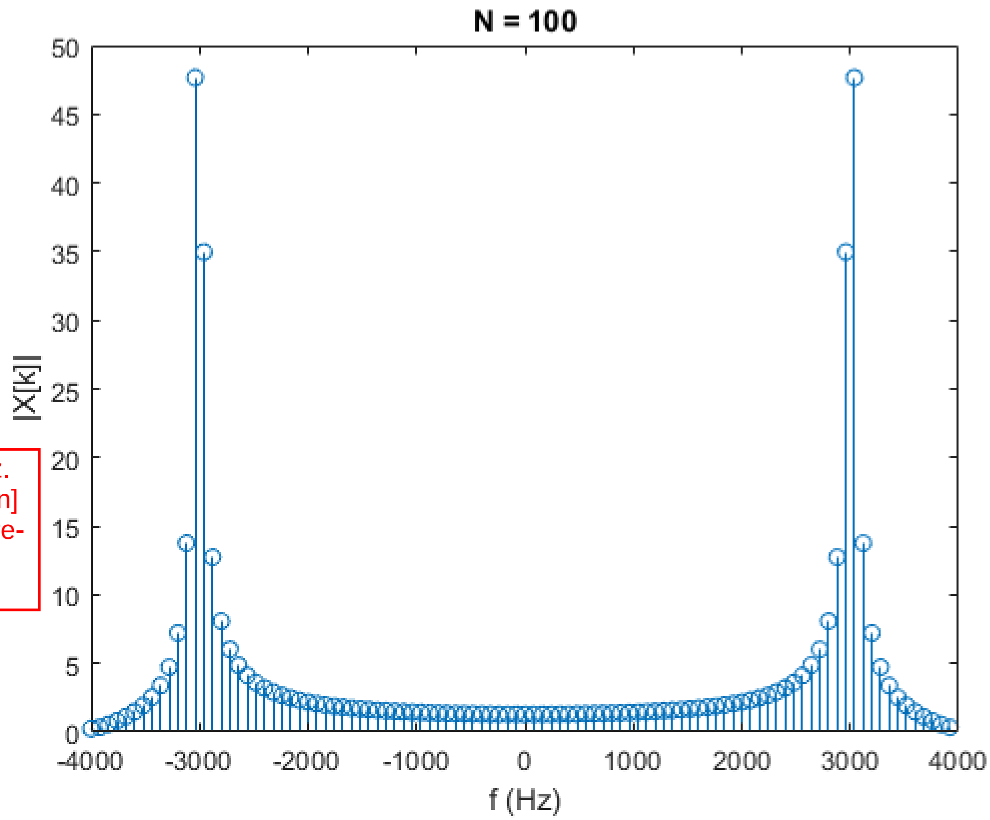
```
N = 100;
n = 0:(N-1);
k = 0:(N-1);
Omega = (-N/2:(N/2-1))*2*pi/N;
f = Omega*fs/(2*pi);
X = fftshift(fft(x,N));
figure
stem(f,abs(X))
title(['N = 10 (Zero padded to 100)'])
xlabel('f (Hz)')
ylabel('|X[k]|')
```



Resolution here is fs/N = 80 Hz. Since zero-padding is used, the spectrum contains 10x more samples.

N = 100; recompute x[n]

```
N = 100;
n = 0:(N-1);
x = cos(2*pi*3000*(n*Ts))+cos(2*pi*3050*(n*Ts));
k = 0:(N-1);
Omega = (-N/2:(N/2-1))*2*pi/N;
f = Omega*fs/(2*pi);
X = fftshift(fft(x,N));
figure
stem(f,abs(X))
title('N = 100')
xlabel('f (Hz)')
ylabel('|X[k]|')
```

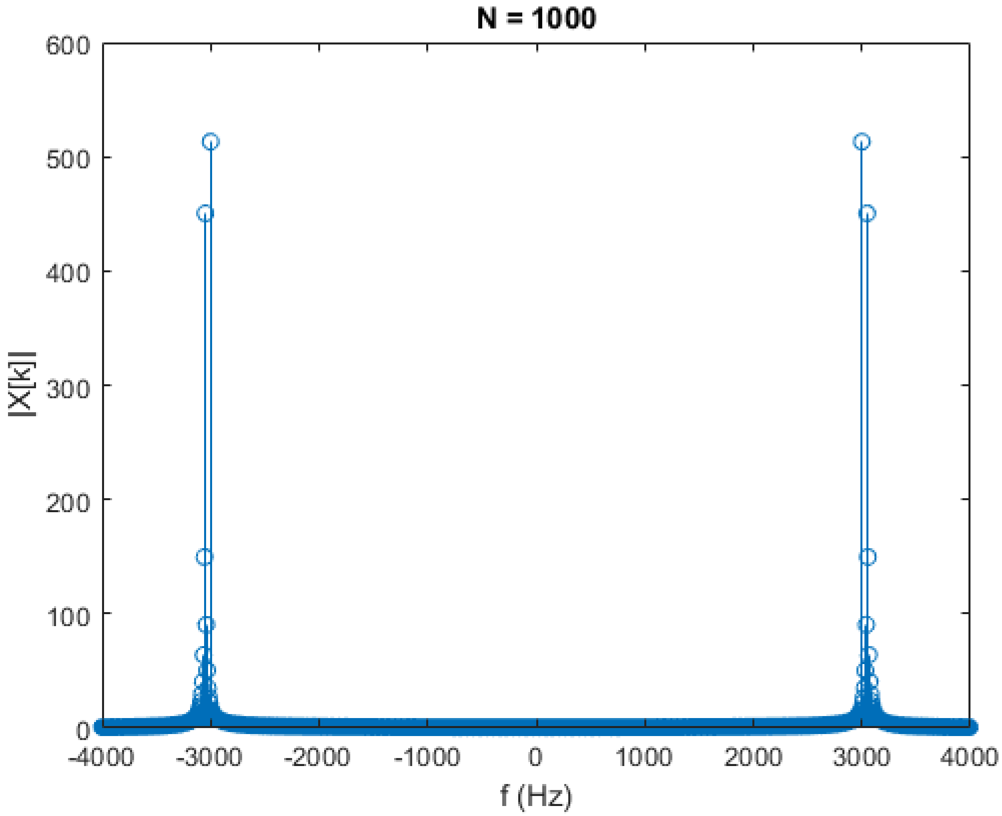


Resolution here is fs/N = 80 Hz. Since 100 actual samples of x[n] were used, the DFT begins to resemble a spectrum containing two impulses.

N = 1000; recompute x[n]

```
N = 1000;
n = 0:(N-1);
x = cos(2*pi*3000*(n*Ts))+cos(2*pi*3050*(n*Ts));
k = 0:(N-1);
Omega = (-N/2:(N/2-1))*2*pi/N;
f = Omega*fs/(2*pi);
X = fftshift(fft(x,N));
figure
stem(f,abs(X))
title('N = 1000')
xlabel('f (Hz)')
ylabel('|X[k]|')
```

DFT now shows two distinct peaks (as expected), with some "leakage" into nearby frequencies.



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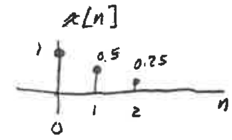
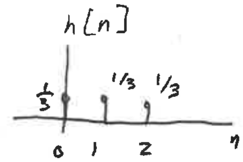
$$x[n] = 0.5^n\big(u[n] - u[n-3]\big)$$
$$= \delta[n] + 0.5\delta[n-1] + 0.25\,\delta[n-2]$$



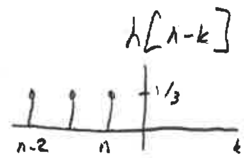
$$h[n] = \tfrac{1}{3}\delta[n] + \tfrac{1}{3}\delta[n-1] + \tfrac{1}{3}\delta[n-2]$$



a) Output will be nonzero for $n=0$ to $n=4$.

$\therefore$ length of output is 5

b)



$n=0$: $y[0] = \frac{1}{3}(1) = 1/3$

$n=1$: $y[1] = \frac{1}{3}\left(\frac{1}{2}\right) + \frac{1}{3}(1) = \frac{1}{2}$

$n=2$: $y[2] = \frac{1}{3}\left(\frac{1}{4}\right) + \frac{1}{3}\left(\frac{1}{2}\right) + \frac{1}{3}(1) = 7/12$

$n=3$: $y[3] = \frac{1}{3}\left(\frac{1}{4}\right) + \frac{1}{3}\left(\frac{1}{2}\right) = \frac{1}{4}$

$n=4$: $y[4] = \frac{1}{3}\left(\frac{1}{4}\right) = \frac{1}{12}$

$$y[n] = \tfrac{1}{3}\delta[n] + \tfrac{1}{2}\delta[n-1] + \tfrac{7}{12}\delta[n-2]$$
$$+ \tfrac{1}{4}\delta[n-3] + \tfrac{1}{12}\delta[n-4]$$

(continued)

$$X(z) = 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}$$

$$H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}$$

$$Y(z) = H(z)X(z)$$

$$= \left(\frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}\right)\left(1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}\right)$$

$$= \frac{1}{3} + \frac{1}{6}z^{-1} + \frac{1}{12}z^{-2} + \frac{1}{3}z^{-1} + \frac{1}{6}z^{-2} + \frac{1}{12}z^{-3} + \frac{1}{3}z^{-2} + \frac{1}{6}z^{-3} + \frac{1}{12}z^{-4}$$

$$= \frac{1}{3} + \frac{1}{2}z^{-1} + \frac{7}{12}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{12}z^{-4}$$

$$y[n] = \mathcal{Z}^{-1}\{Y(z)\}$$

$$= \frac{1}{3} + \frac{1}{2}\delta[n-1] + \frac{7}{12}\delta[n-2] + \frac{1}{4}\delta[n-3] + \frac{1}{12}\delta[n-4]$$

DTFT calculation follows the same approach as Z-transform.

$$Y(e^{j\omega}) = \underbrace{\left(\frac{1}{3} + \frac{1}{3}e^{-j\omega} + \frac{1}{3}e^{-j2\omega}\right)}_{H(e^{j\omega})}\underbrace{\left(1 + \frac{1}{2}e^{-j\omega} + \frac{1}{4}e^{-j2\omega}\right)}_{X(e^{j\omega})}$$

$$= \frac{1}{3} + \frac{1}{2}e^{-j\omega} + \frac{7}{12}e^{-j2\omega} + \frac{1}{4}e^{-j3\omega} + \frac{1}{12}e^{-j4\omega}$$

$$1 \xleftrightarrow{\mathcal{F}} \delta[n] \quad \text{and} \quad x[n-N] \xleftrightarrow{\mathcal{F}} e^{-j\omega N}X(e^{j\omega})$$

$$\therefore y[n] = \frac{1}{3} + \frac{1}{2}\delta[n-1] + \frac{7}{12}\delta[n-2] + \frac{1}{4}\delta[n-3] + \frac{1}{12}\delta[n-4]$$

in MATLAB:

```
x = [1 0.5 0.25];
h = [1/3 1/3 1/3];

X = fft(x,5);        % use 5pt FFT for linear convolution
H = fft(h,5);

Y = H.*X;

y = ifft(Y);
```