EE3221 Homework 8
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Assigned: 6 May 2020
No submittal required. No quiz.

Note: Problems from the course textbook (Ulaby and Yagle, 2018) are specified with the prefix "UY".

1. Consider the signal $x[n] = \delta[n]$. Find the DTFT of $x[n]$. Now, compute the 10 point DFT of $x[n]$. Explain how the two results relate to one another.
   ANSWERS: $X(e^{j\Omega}) = 1$. $X[k] = 1$ for $k = 0, 1, \ldots, 9$.

2. Consider the discrete-time sequence:

$$x[n] = \{3, 4, 1, 2\}.$$

   (a) Compute $X[k]$ using the DFT equation.

   (b) Compute $X[k]$ using the radix-2 FFT algorithm. Show each step of the algorithm.

   ANSWERS: $X[k] = \{10, 2 - j2, -2, 2 + j2\}$.

3. Suppose the following analog signal is sampled using $f_s = 8\text{kHz}$ to create the sequence $x[n]$.

$$x(t) = \cos(2\pi 3000t) + \cos(2\pi 3050t).$$

   For each of the following, write a MATLAB script that computes and plots the requested DFTs. For these calculations:

   - Use MATLAB's `fft()` function to compute the DFTs.
   - Plot the magnitude of the DFTs using a "stem" style plot. Label the frequency axis in Hz. Include MATLAB code and the resulting plot.
   - For an N point DFT, note that the DFT resolution is the spacing between each frequency bin, which is f_s/N.

   (a) Compute and plot a 10 point DFT of $x[n]$.

   (b) Repeat the previous problem, but zero pad the 10 samples of $x[n]$ so as to perform a 100 point DFT. That is, compute the DFT of a vector containing the 10 original samples followed by 90 zeros.

   (c) Compute and plot a 100 point DFT, but now use 100 actual samples of $x[n]$.

   (d) Compute and plot a 1000 point DFT, again using 1000 actual samples of $x[n]$.

   (e) Provide a brief summary of what is being seen (or not seen!) in each plot. Explain the results.

4. Consider the signal

$$x[n] \quad = \quad 0.5^n(u[n] - u[n-3])$$

   that is the input to the discrete-time LTI system with impulse response

$$h[n] \quad = \quad \frac{1}{3}(u[n] - u[n-3])$$

   It is possible to compute the output signal $y[n]$ using the DFT, just as was done with other frequency domain techniques. However, one must take care to ensure the approach is equivalent to *linear convolution* rather than *cyclic convolution*.

   IMPORTANT: Please review UY 7-15.4 through 7-15.6 before continuing.

(a) Determine the range of n values for which $y[n]$ will be non-zero.

(b) Compute $y[n]$ using an approach previously studied in the course. You may use (i) time-domain convolution, (ii) the DTFT convolution property, or (iii) the Z-transform convolution property.

(c) Write a MATLAB script that computes y[n] using the DFT. You may use the `fft()` and `ifft()` functions. However, be sure you size the DFT calculation so that it results in linear convolution!