Name: Solution

EE3221 Digital Signal Processing
Homework/Quiz 6
Dr. Prust

Homework Score	/ 5
Quiz Score	/ 5
Total	/ 10

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1. Consider a discrete-time LTI system having the following difference equation:

$$y[n] = x[n] - 0.5x[n-1]$$

(a) Find the poles and zeros of the system.

$$Y(z) = X(z) - 0.5z^{-1}X(z)$$
$$= X(z)\left[1 - 0.5z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 - 0.5z^{-1} = \frac{z - 0.5}{z}$$

$\therefore$ zero at $z = 0.5$
pole at $z = 0$

(b) Find the system frequency response $H(e^{j\Omega})$.

$$H(e^{j\Omega}) = H(z)\Big|_{z=e^{j\Omega}} = 1 - 0.5e^{-j\Omega}$$

(c) Suppose the system input $x[n]$ has fundamental period $N_0 = 5$ and discrete-time Fourier series coefficients $\mathrm{x}_0 = 2$, $\mathrm{x}_1 = 0.75$, and $\mathrm{x}_2 = -1$. All other coefficients are zero. Find the discrete-time Fourier series coefficients for the output signal $y[n]$. Simplify the coefficients as much as possible.

$$\Omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{5} \qquad y_k = H(e^{jk\Omega_0})x_k$$

$$y_0 = H(e^{j0})x_0 = (1 - 0.5)2 = 1$$
$$y_1 = H(e^{j\frac{2\pi}{5}})x_1 = \left(1 - 0.5e^{-j2\pi/5}\right)0.75 = 0.73e^{j0.51} = 0.63 + j0.36$$
$$y_2 = H(e^{j2\frac{2\pi}{5}})x_2 = \left(1 - 0.5e^{-j4\pi/5}\right)(-1) = 1.43e^{-j2.94} = -1.40 - j0.29$$