The Effect of an Arterial Catheter on Hemodynamics (v. 1.0)

Computer Project 1

BE-382, Winter 2008-2009

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Due Thursday, January 22 by 4pm. You may work in pairs or alone on this project.

This project was partially inspired by the eliminated senior design project from your class. The goal is to determine the effects of an arterial catheter on hemodynamics and to estimate the drag force from flowing blood on such a catheter.

Introduction: Arterial Catheterization

Arterial catheters are commonly used to help diagnose and treat problems associated with blockages of the coronary arteries. During catheterization, a catheter is inserted into the femoral artery (in the groin) and then fed retrograde through the aorta. Just distal to the aortic valve are the entrances (ostia) to the left and right coronary arteries. Under the guidance of fluoroscopy, the catheter is fed into one of these arteries. For diagnosis of blockages in the coronary arteries, a contrast dye can be injected directly into one of these vessels using such a catheter, and a fluoroscope can be used to visualize the 3-D shape of the lumen of the artery to determine if any stenoses (blockages) are present. Alternatively, these catheters can be used to deliver stents that are opened using a balloon at the tip of the catheter.

Part 1: Analytical Solution

Use the Navier-Stokes equations to show that the velocity profile for fully-developed pressure-driven flow between two stationary, concentric cylinders is given by the equation below. Assume that the inner cylinder has a radius of R_1 and that the outer cylinder has a radius of R_2. The axial pressure drop is constant and given by $\Delta P/L$, and the fluid is Newtonian with a viscosity of μ. To receive full credit, you must follow the solution procedure outlined in the course notes. Your final answer should be an equation solved for axial velocity, v_z, as a function of radial position, r, in terms of the constants given. The continuity and Navier-Stokes equations for cylindrical coordinates are given on the last page of this assignment. Submit this page to demonstrate the elimination of terms.

$$v_z = \frac{1}{4\mu}\frac{\Delta P}{L}\left[r^2 - R_1^{\,2} + \frac{\left(R_1^{\,2} - R_2^{\,2}\right)\ln\left(r/R_1 \right)}{\ln\left(R_2/R_1 \right)} \right]$$

Part 2: Plotting the profiles for constant ΔP/L

Suppose such a catheter is inserted into an aorta in which the pressure drop (Δ*P/L*) is −4.00 g/(cm^2s^2). The blood viscosity is 0.027 g/(cm s). Use Excel to plot v_z as a function of *r* in an aorta with a radius (R_2) of 1 .00cm for each of the three following situations:

(a) in the absence of a catheter using the standard pipe flow equation:

$$\boldsymbol{v}_z = -\frac{\Delta \boldsymbol{P}}{\boldsymbol{L}} \frac{\boldsymbol{R}_2^{\;2}}{4\mu}\left(1 - \frac{\boldsymbol{r}^2}{\boldsymbol{R}_2^{\;2}}\right)$$

(b) in the presence of a catheter with a radius (R_1) of 0.020 cm placed in the center of the aorta,
(c) in the presence of a catheter with a radius (R_1) of 0.100 cm placed in the center of the aorta.

In your spreadsheet, use 0-cm to 1-cm as your range for *r*, with a spacing of 0.010 cm. For parts (b) and (c), only plot the points between R_1 and R_2. *Show all three plots on the same graph*. Note that a catheter of 0.100 cm radius (6 French) is typical for clinical use.

Part 3: Estimation of flow rate reduction for constant ΔP/L

Determine the percentage reduction in flow rate caused by the presence of the catheters, assuming Δ*P/L* is constant at the value given in Part 2. To do this, use simple numerical integration. Flow rate is found by integrating velocity over the cross-sectional area. Recall that for cylindrical coordinates, circumferential integration requires us to multiply $d\theta$ by *r*. Therefore, we can get something proportional to flow rate by multiplying each velocity by its respective radius and then adding them all up. Do this for each of the three cases above (remembering to only integrate between R_1 and R_2 for the catheter situations). We've been sloppy with units here, but we can cancel them out by expressing the flow rates for the 0.020 cm and 0.100 cm catheters as a percentage of the flow rate without a catheter. Please calculate these two numbers.

Part 4: Analysis for constant flow rate

In reality, the fluid resistance of the aorta is very small compared to the downstream vessels, and the pressure drop across the aorta is small. Thus, when the resistance of the aorta increases, as happens in the presence of a catheter, the network resistance is minimally impacted, and the flow rate remains relatively constant. Thus, it would be more accurate to force the flow rate to be constant and allow Δ*P/L* to change.

Though an analytical solution would be relatively simple, let's approach the problem numerically instead. *Perform the following integrations in separate columns so that you do not destroy the plots you made earlier*. In Part 3, you numerically integrated rv_z over $R_1 \rightarrow R_2$ to get something proportional to *Q* for both the uncatheterized and catheterized cases. Your expression for rv_z should make reference to a cell containing Δ*P/L*. For each of the two catheter sizes, use Excel's Solver to adjust Δ*P/L* such that the flow rate with the catheter approaches that of the uncatheterized aorta. This can be done by minimizing a cell containing $(Q_{cath}-Q_{uncath})^2$.

Create a new graph similar to that created in Part 3, but now for constant Q. Once again, the plot should contain three velocity profiles. Report your $\Delta P/L$ for each of the two catheters.

Part 5: Determination of the force acting on the catheter

The flowing blood imparts a drag force on the stationary catheter in the aorta. If the catheter were to be advanced, this drag force would have to be exceeded. We can calculate the drag force by determining the shear stress acting on the catheter and then multiplying this by the surface area of the catheter.

Use the velocity profile from Part 1 to derive an analytical expression for shear stress. Using the $\Delta P/L$ values you calculated in Part 4, calculate the shear stress (in g/(cm s^2)) on each of the catheters under the constant Q assumption. Use these shear stresses to calculate the drag force (in N) on each of the two catheters assuming the length of the catheter in the aorta is 50.0 cm.

We can also calculate a numerical approximation for dv_z/dr and use it to calculate the shear stress. This can be done by calculating $\Delta v_z/\Delta r$ using the velocity at the point closest to the catheter wall. This is the method used to calculate wall shear stresses for computational fluid dynamics (CFD) solutions when analytical solutions are not available. Do this for each catheter using the data in your Excel spreadsheet from Part 4. Calculate the resulting shear stresses and compare them to the analytical solutions. What might account for the discrepancies?

Report submission requirements

Compose a brief report in which you detail the various analyses you performed. For each part, include a brief description of what you did, your results, and any interesting observations you made. Be sure to address any specific points made in the assignment. Embed your Excel graphs in the report. Any handwritten parts can be placed in appendices at the end of the report, providing they are properly referenced within the report.

A few additional discussion points:

- Discuss which of the two catheters had the larger drag force acting on it and why.
- Hypothesize what the effect would be if the catheter were not centered in the aorta. What effect might this have on the flow profile and drag force?
- Discuss any special issues that would arise if the catheter were placed in a very small vessel with a diameter only slightly larger than that of the catheter.

Submit your memo and Excel spreadsheet to me via e-mail.

Cylindrical Continuity Equation and Navier-Stokes Equations

$$\frac{\partial \rho}{\partial t} = -\left(\frac{1}{r} \frac{\partial (\rho r v_r)}{\partial r} + \frac{1}{r} \frac{\partial (\rho v_\theta)}{\partial \theta} + \frac{\partial (\rho v_z)}{\partial z} \right)$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial (r v_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = -\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial (r v_\theta)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho g_z$$