

## Assignment 6

Part A Solution

1)

Given: transport of LDL across endothelium.

$$k = 10^{-8} \text{ cm/s}$$

$$C_{\text{blood}} = 100 \text{ mg/dL} = 1 \text{ mg/cm}^3$$

$$C_{\text{wall}} = 0 \text{ mg/cm}^3$$

Find:  $N_{\text{LDL}}$  over a 0.5-cm diameter, 2-cm length tube over 24 hr.

Assume: steady state  
no convective transport  
electrically neutral

Solution. This is simple 1-D membrane transport by diffusion only. Therefore,

$$J = k \Delta C = (10^{-8} \text{ cm/s})(1 \text{ mg/cm}^3 - 0 \text{ mg/cm}^3) = 10^{-8} \text{ mg/cm}^2 \cdot \text{s}$$

$$N = J \cdot A \cdot \Delta t = (10^{-8} \text{ mg/cm}^2 \cdot \text{s}) \left( \pi \cdot (0.5 \text{ cm}) \cdot (2 \text{ cm}) \right) \left( 24 \text{ hr} \right) \left( \frac{3600 \text{ s}}{1 \text{ hr}} \right)$$

$$N = 0.0027 \text{ mg} = \boxed{2.7 \mu\text{g}}$$

Risk factors:

- hypercholesterolemia (High LDL in blood)

$$\rightarrow \uparrow C_{\text{blood}} \rightarrow \uparrow \Delta C \rightarrow \uparrow J \rightarrow \uparrow N$$

- smoking, diabetes, irregular blood flow patterns all affect the endothelial cells, making them extra leaky to LDL.

$$\rightarrow \uparrow k \rightarrow \uparrow J \rightarrow \uparrow N$$

2) a) Let  $k$  be the permeability of  $\text{Na}^+$ .

→ permeability of  $\text{K}^+$  =  $10k$

→ permeability of  $\text{Cl}^-$  =  $xk$

Substituting into expanded Nernst equation, with  $V_m = -86\text{mV}$   
and  $RT/F = 26.72\text{mV}$ ,

$$-86\text{mV} = 26.72\text{mV} \ln \frac{10k[\text{K}^+]_o + k[\text{Na}^+]_o + xk[\text{Cl}^-]_i}{10k[\text{K}^+]_i + k[\text{Na}^+]_i + xk[\text{Cl}^-]_o}$$

( $i$  = inside cell,  $o$  = outside cell)

Substituting in given values of concentration,

$$-86\text{mV} = 26.72\text{mV} \ln \frac{10k(2.5\text{mM}) + k(122\text{mM}) + xk(2\text{mM})}{10k(140\text{mM}) + k(9\text{mM}) + xk(122\text{mM})}$$

$$-3.219 = \ln \frac{25 + 122 + 2x}{1400 + 9 + 122x} = \ln \frac{147 + 2x}{1409 + 122x}$$

$$0.040 = \frac{147 + 2x}{1409 + 122x}$$

$$56.4 + 4.88x = 147 + 2x$$

$$2.88x = 90.6$$

$$\boxed{x = 31.5} \Rightarrow \text{permeability of } \text{Cl}^- \text{ is } 31.5x \text{ that of } \text{Na}^+.$$

b) very large  $\text{Na}^+$  permeability makes permeability of  $\text{Cl}^-$  &  $\text{K}^+$  negligible, so

$$V_m = (26.72\text{mV}) \ln \frac{[\text{Na}^+]_o}{[\text{Na}^+]_i} = 26.72\text{mV} \ln \frac{122\text{mM}}{9\text{mM}} = \boxed{69.7\text{mV}}$$

For  $\text{K}^+$ :

$$V_m = (26.72\text{mV}) \ln \frac{[\text{K}^+]_o}{[\text{K}^+]_i} = 26.72\text{mV} \ln \frac{2.5\text{mM}}{140\text{mM}} = \boxed{-107.6\text{mV}}$$

For  $\text{Cl}^-$ :

$$V_m = (26.72\text{mV}) \ln \frac{[\text{Cl}^-]_i}{[\text{Cl}^-]_o} = 26.72\text{mV} \ln \frac{2\text{mM}}{122\text{mM}} = \boxed{-109.8\text{mV}}$$