Solution

Biomedical Differential Component Mass Balance MATLAB PDE Toolbox Problem
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This problem demonstrates how to use the MATLAB Partial Differential Equation (PDE) Toolbox to find the steady-state oxygen concentration distribution a wedge shaped region of tissue. I recommend you work through this handout at a computer while running Matlab. The region of interest consists of a 2-dimensional, wedge shaped area of tissue as shown in Figure 1.

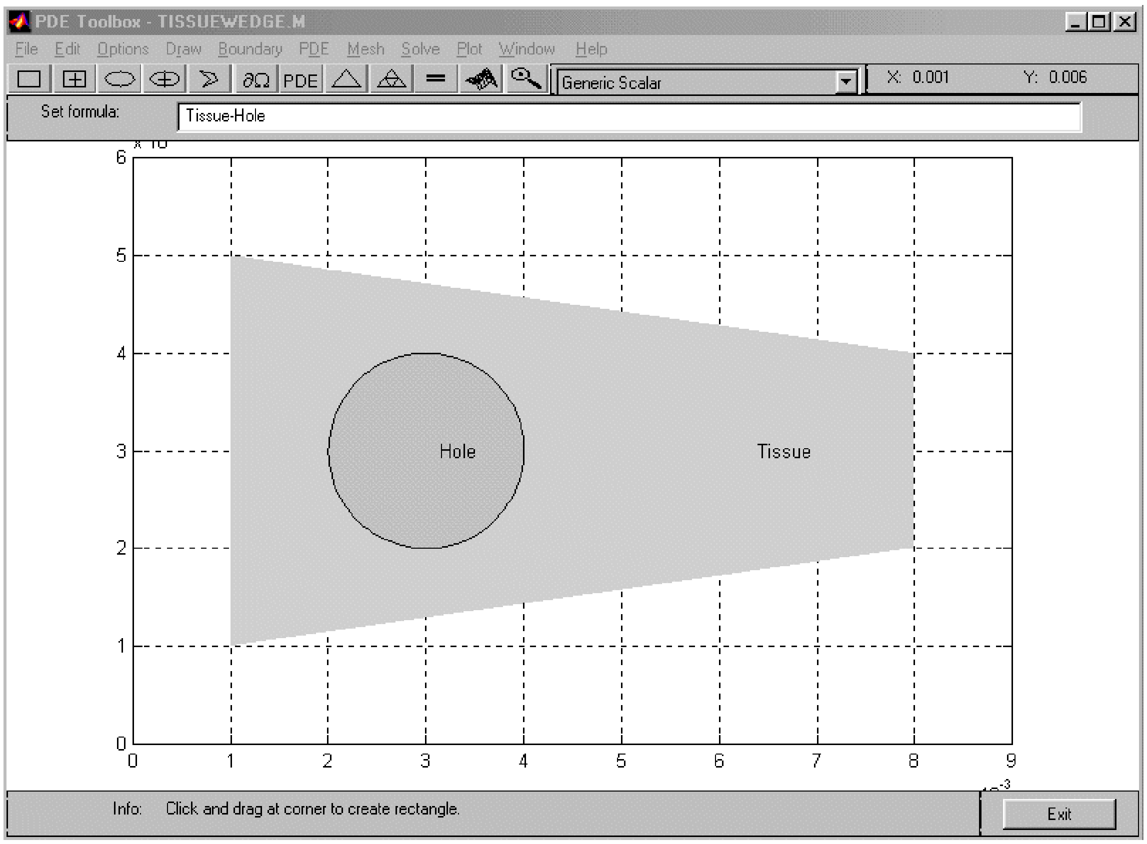

Figure 1: PDE Tool Draw Mode screen showing region of interest.

The region consists of the wedge of tissue $7.00\text{x}10^{-3}$ cm long by $4.00\text{x}10^{-3}$ cm wide at its wider end and $2.00\text{x}10^{-3}$ cm wide at its narrower end. It has a $2.00\text{x}10^{-3}$ cm diameter hole (or impervious inclusion) located as shown. The left and right hand ends of the wedge are assumed impervious to oxygen. An artery runs along its upper edge while a vein runs along its lower edge. The artery keeps the oxygen concentration at 128 µM everywhere along the upper edge while the vein keeps the oxygen concentration at 54.0 µM everywhere along the lower edge. These values correspond to oxygen partial pressures of 95.0 and 40.0 mm Hg in the artery and vein, respectively, and a Henry's Law coefficient of 0.74 mm Hg/µM in the tissue. The diffusivity of oxygen in the tissue is assumed to be $6.3\text{x}10^{-6}$ cm/s^2 and the metabolic consumption rate is assumed to be 50.0 µmoles/(L·s).

The following steps were executed to specify the problem geometry in MATLAB. First, start MATLAB's PDE Toolbox by entering *pdetool* at the **MATLAB** prompt. The *Options | Axes Limits* menu choice was used to specify axes from 0 to 0.009 cm for *x* and 0 to 0.006 cm for *y*. These values account for the 3 to 2 aspect ratio of the drawing area and provide a nicely scaled drawing. The *Option* menu was also used to activate the *Grid* and *Snap* features to make drawing easier. The tissue region was then drawn using the *Polygon* tool. Double clicking on the region allowed its label to be changed to the word *Tissue*. Next, the hole was specified using the *Centered Circle* tool and labeled *Hole*. Finally, the *Set Formula* line was changed to read *Tissue-Hole* to complete the specification of the problem geometry.

Next, the boundary conditions were specified as follows after selecting the *Boundary Mode*. The shift key was held down and the right and left ends of the tissue wedge and 4 quadrants of the circular hole were selected using the mouse. The *Boundary | Specify Boundary Conditions…* menu option was chosen. The *Neumann* type boundary was specified and the quantities *q* and *g* were both set to zero. Next, double clicking on the upper edge of the tissue wedge opened the Boundary Condition dialog for it. A Dirichlet type boundary was specified and *h* set to 1.00 and r set to 128 μM (the oxygen concentration in the tissue in contact with the artery). Double clicking on the lower edge of the tissue wedge opened the Boundary Condition dialog for it. A Dirichlet type boundary was specified and *h* set to 1.00 and r set to 54.0 μM.

Next, the actual PDE to be solved was specified in the *PDE Mode*. *Double clicking on the region of interest opened PDE Specification dialog*. An *Elliptic* type PDE was specified and *c* set to 6.3×10^{-6} cm/s^2 (the diffusivity of oxygen in the tissue), *a* set to 0 and *f* set to -50.0 μmole/(L·s) (the metabolic oxygen **generation** rate).

Meshing the geometry by clicking on the *Initialize Mesh* button completed the solution. The *Refine Mesh* button was used once to improve the quality of the result and the *Solve PDE* (=) button was clicked to calculate the concentration distribution. The *Plot | Parameters…* menu choice was then used to specify the appearance of the results. Figure 2 shows a *Color* plot of *u* (concentration) using the *jet* colormap. It looks best in color. Figure 3 shows a *Contour* plot of *u* using the *jet* colormap. It looks best when printed from within MATLAB in *Landscape* orientation, centered and with colored lines and text printed in *Black and White*. The direction and relative magnitude of the oxygen flux can be seen in Figure 4. This is an *Arrows* plot of –grad(*u*) in *proportional* style.

References

1. Anonymous. On-line MATLAB documents. Release 11.
2. Anonymous. MATLAB Partial Differential Equation Toolbox User Guide, Version 1. The MathWorks, Inc. 1997.
3. Fournier, R. L. "Basic Transport Phenomena in Biomedical Engineering." Taylor & Francis 1999

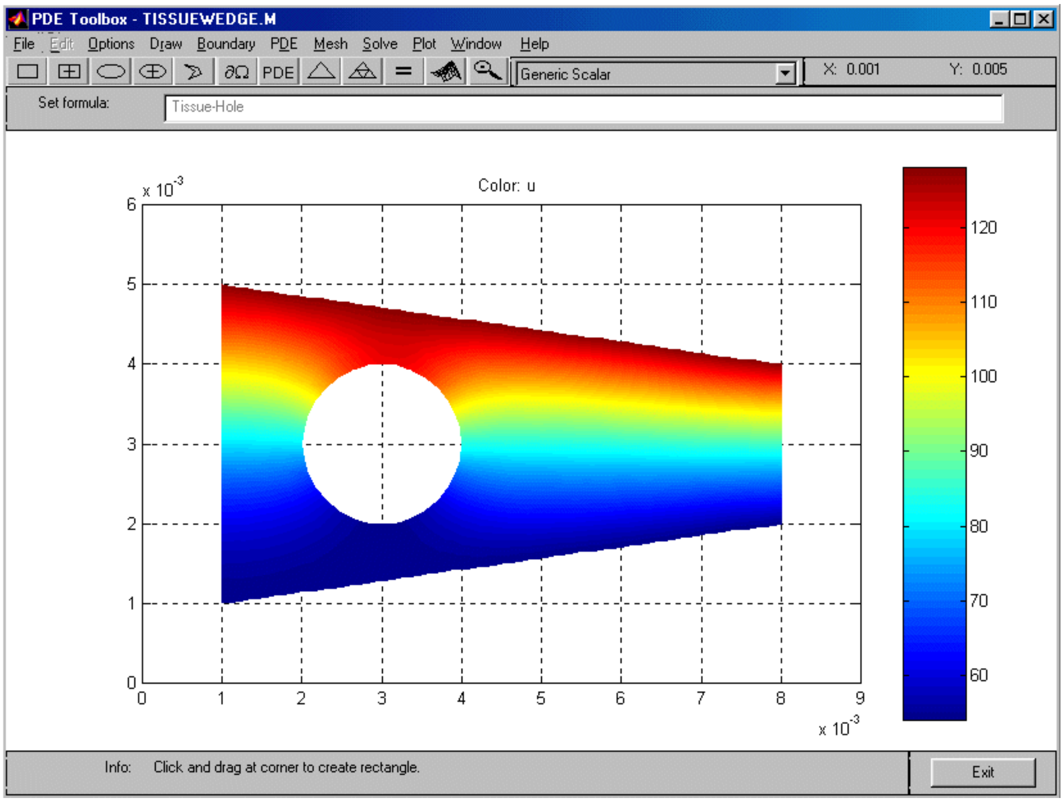

Figure 2: Color plot of concentration distribution.

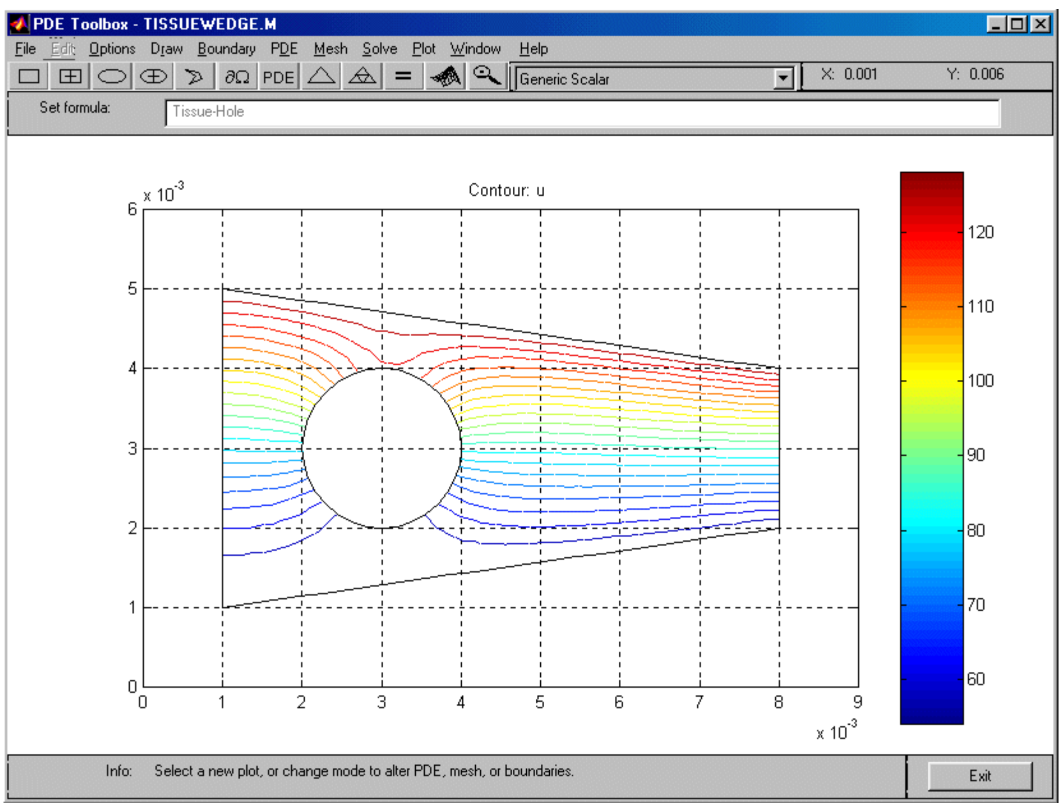

Figure 3 Contour plot of concentration distribution.

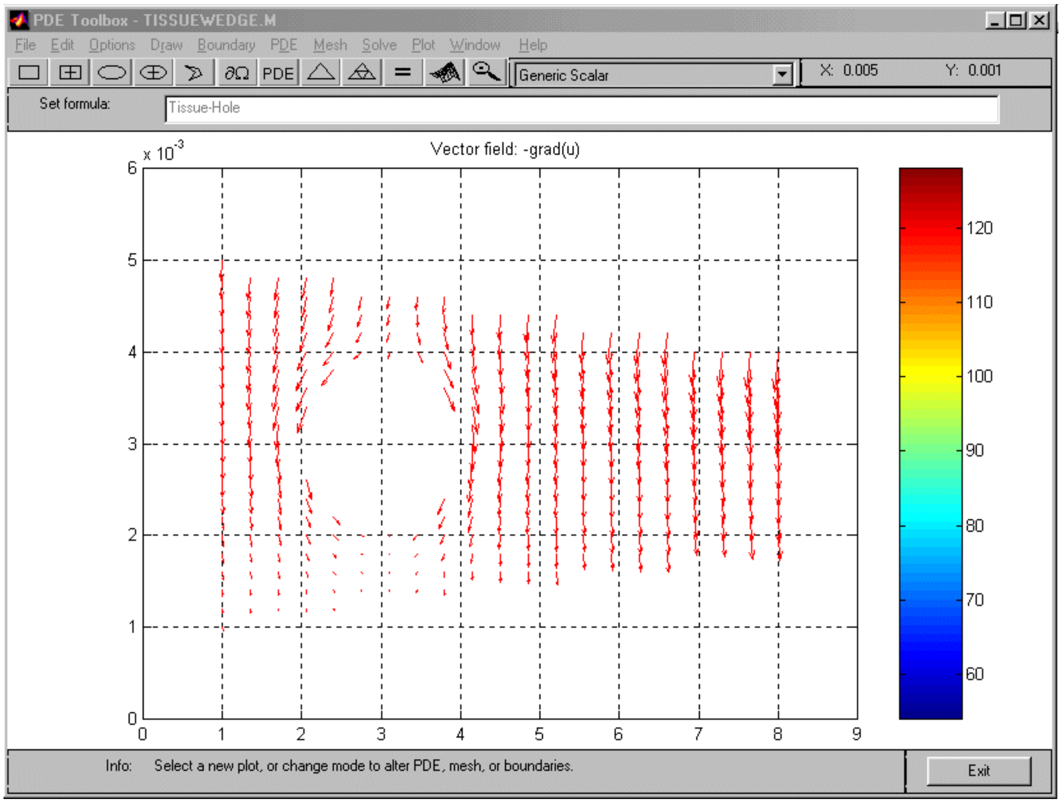

Figure 4 Arrow plot showing oxygen flux.