Exam 2 – Key – v. 1.5 (Ave 55, High 72, Low 20 (☹))
BE-382, Winter '08-'09, Dr. C. S. Tritt

1. (30) In an attempt to reduce the overall pressure drop in a 20.0 m long, 1.00 cm diameter cooling water line, a 16.0 m long segment in the middle of the line is replaced with a 2.00 cm diameter tube. Estimate and compare the pressure drops in these two configurations. The volumetric flow rate is 2.00×10^{-4} m^3/s (which is equivalent to 12.0 liters/min). The density of the water is 1,002 kg/m^3 and viscosity is 1.00×10^{-3} kg/(m·s) (which is equal to 1.00 cp). Assume smooth surfaces, fully developed flow and an energy correction factor, α, of 1.00.

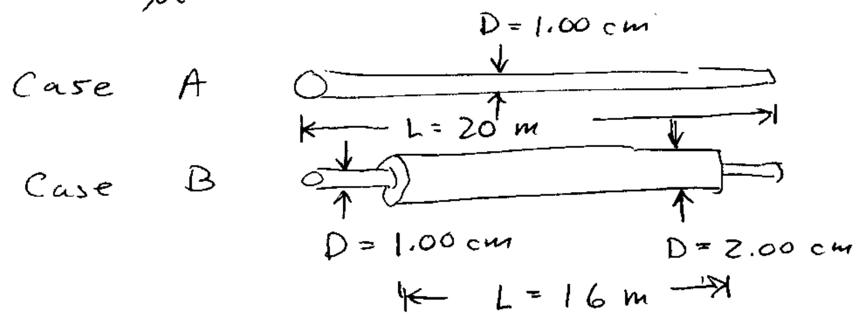
1. Common values (given)

$\dot{V} = 2.00 \times 10^{-4}$ m^3/s

$\rho = 1{,}002$ kg/m^3

$\mu = 1.00 \times 10^{-3}$ $kg/m \cdot s$

Assume $\alpha = 1$



Values in segments

D (m)	0.01	0.02
$A = \frac{\pi D^2}{4}$ (m^2)	7.85×10^{-5}	3.14×10^{-4}
$v = \dot{V}/A$ (m/s)	2.55	0.637
$N_{Re} = \frac{\rho v D}{\mu}$ (none)	25,500	12,800
f (from chart)	0.026	0.028

"Minor Losses"

Expansion $K_{L,e} = \alpha \left(1 - \frac{d^2}{D^2}\right)^2 = \left(\frac{3}{4}\right)^2 = 0.562$

Contraction $K_{L,c} = 0.400$ (at $\frac{d^2}{D^2}$ of 0.25)

$$\Delta P_{Pipes} = f \frac{L}{D} \frac{\rho V^2}{2} \quad 4 \text{ pts}$$

$$\Delta P_L = K_L \frac{\rho V^2}{2} \quad 4 \text{ pts}$$

ΔP's for the two Cases:

Case I: $D = 1.00$ cm, $L = 20$ m

$$\Delta P_I = 1.69 \times 10^5 \text{ Pa} = \underline{\underline{169 \text{ KPa}}} \quad 10 \text{ pts}$$

−6 pts if not determined.

Case 2: $D = 2.00$ cm, $L = 16$ m +
$D = 1.00$ cm, $L = 4$ m +
sudden Expansion +
sudden contraction

$$\Delta P = 4.55 \times 10^3 + 3.39 \times 10^4 + 1.83 \times 10^3 + 1.30 \times 10^3$$
$$= 4.16 \times 10^4 \text{ Pa} = \underline{\underline{41.6 \text{ KPa}}} \quad \mathbf{12} \text{ pts}$$

So, the pressure drop is less in case 2.

Most wrong −25

Correct K_L's → minor loss ΔP's <u>only</u> −20

w/ Some correct pipe drops −10

2. (35) International rules state that ping-pong (table tennis) balls shall weigh 2.7 grams and be 40 millimeters in diameter. What is the terminal velocity of a ping-pong ball at sea level where μ_{air} = 1.82×10^{-5} kg/(m·s), ρ_{air} = 1.20 kg/m^3 and g = 9.81 m/s^2? Neglect buoyancy (I assume it's accounted for in the specified weight of the ball). Without doing any calculations, would a solid depleted uranium (ρ = 19.1 g/cm^3) ball of the same diameter have a higher or lower terminal velocity under the same conditions?

1. Common values (given)

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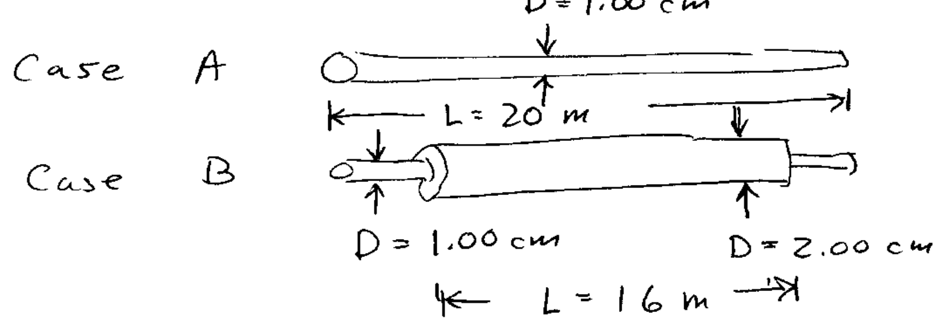
$\mu = 1.00 \times 10^{-3}$ kg/m·s



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Iteration

Guess V (m/s)	N_{Re}	C_D	F_D
1	2630	0.4	3.01×10^{-4}
10	26,300	0.4	3.01×10^{-2}
9.0	23,700	0.4	2.44×10^{-2}

At this point, I know $9.0 < v < 10$ m/s
In general, I could keep guessing until I found the answer. However, in this case C_D is not changing, so just solve $F_g = \frac{A\rho}{2} C_D v^2$ for v to get:

$$v = 9.37 \text{ m/s}$$

Depleted uranium would have a higher terminal velocity because the same drag equations would apply but F_g would be much greater.

For depleted uranium, I get $V = 332$ m/s

C_D off by 10x −2
Not showing calc. of $N_{Re} \Rightarrow C_D$ −8
Using Stoke's Law - not checking result −20

3. (35) Cross out terms on the given differential balance equations and provide boundary and initial conditions as needed to solve for the steady state velocity distribution in the fully developed flow of an incompressible fluid in a square duct with dimensions 2W wide by 2H high. Align the direction of flow with the z-axis of your coordinate system. Some credit will be given for choosing the best origin for your coordinate system.

Continuity –

$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\overset{1}{\cancel{\frac{\partial u_z}{\partial z}}}=0$$

Rectangular (Cartesian) Coordinates –

x direction

$$\rho\left(\overset{2}{\cancel{\frac{\partial v_x}{\partial t}}}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+\overset{1}{\cancel{v_z\frac{\partial v_x}{\partial z}}}\right)=-\overset{3}{\cancel{\frac{\partial p}{\partial x}}}+\mu\left[\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\overset{1}{\cancel{\frac{\partial^2 v_x}{\partial z^2}}}\right]+\overset{4}{\cancel{\rho g_x}}$$

y direction

$$\rho\left(\overset{2}{\cancel{\frac{\partial v_y}{\partial t}}}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+\overset{1}{\cancel{v_z\frac{\partial v_y}{\partial z}}}\right)=-\overset{3}{\cancel{\frac{\partial p}{\partial y}}}+\mu\left[\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\overset{1}{\cancel{\frac{\partial^2 v_y}{\partial z^2}}}\right]+\overset{4}{\cancel{\rho g_y}}$$

z direction

$$\rho\left(\overset{2}{\cancel{\frac{\partial v_z}{\partial t}}}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+\overset{1}{\cancel{v_z\frac{\partial v_z}{\partial z}}}\right)=-\overset{3}{\cancel{\frac{\partial p}{\partial z}}}+\mu\left[\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\overset{1}{\cancel{\frac{\partial^2 v_z}{\partial z^2}}}\right]+\overset{4}{\cancel{\rho g_z}}$$

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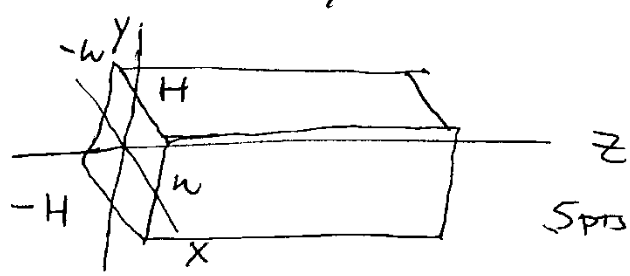
3. Apply continuity and Navier-Stokes equations.

I.C.'s not required for steady-state problems. 5 pts

Coordinate system



5 pts

B.C's

No slip $\begin{cases} u_z = u_x = u_y = 0 & @ \ x = W \ \& \ -W \\ u_z = u_x = u_y = 0 & @ \ y = H \ \& \ -H \end{cases}$

5 pts

sym.

sym.

$$\left\{ \begin{array}{l} \dfrac{\partial u_z}{\partial x} = 0 \ @ \ x = 0 \\ \dfrac{\partial u_z}{\partial y} = 0 \ @ \ y = 0 \\ u_x = 0 \ @ \ x = 0 \\ u_y = 0 \ @ \ y = 0 \end{array} \right\}$$

5 pt

Elimination of terms

1) Fully developed implies no changes in the z-direction 5pts

2) Steady state implies no changes with time 5pts

3) Neglect pressure changes
4) Neglect gravity

Note if continuity is ~~left~~ in, not many other terms can be eliminated. If it is used to claim $u_y + u_x = 0$ everywhere, terms containing these can be eliminated. However, you can't use both and must explicitly state continuity was used.

Need to comment regarding inclusion of continuity (2pts)

Incompressible implies ρ is constant so used incompressible version of continuity.