Exam 3 Key (Initial – ave 75, low 61, high 98.)
BE-382, Winter '08-'09, Dr. C. S. Tritt

This is a 1½ hour closed book, closed notes exam. Write your answers on the paper provided or on your own paper. Organize and explain you work for full and partial credit. You may use 1 double sided, 8½ by 11 inch equation sheet, EES and the provided Units Conversion Factors attachment. If you use EES, e-mail me your files (the file names should encode your last name and the problem number) at the end of the exam and clearly indicate on your exam that you used EES. There are 3 problems, point values are as indicated.

1. (10) Describe and give an example of a Neumann and a Dirichlet boundary condition related to either DCMB or DEB problems.

   Dirichlet B.C.'s specify the value of the quantity of interest (velocity, concentration or temperature) at the boundary. Examples include fast chemical reactions that keep surface concentrations zero and boiling (or condensing) liquids, that often, but not always, results in a fixed surface temperature (due to effectively infinite convective coefficient).

   Neumann B.C.'s specify the value of the gradient of the quantity of interest at the boundary. This gradient can be zero (for example at an impenetrable or ideally insulated surface or a point, line or plane of symmetry) or a function of the value quantity at the surfaces (as it is in cases of convection). A special type of Neumann boundary condition is the Robin boundary condition. In Robin B.C.'s the gradient is specified as a function of the value of the quantity of interest. Robin B.C.'s typically occur in cases of convection. Discussion of Robin B.C.'s was not required for full credit. Examples are woven into the discussion above.

   Reversing the names was −7 (nobody did this). Saying Neumann B.C.s involve specification of the flux and not mentioning the gradient was −1 (the flux the result of the gradient, but it is the gradient that has the same terms in it as the PDE and, therefore, is what is needed to solve the PDE). Examples were worth 2 pts each. Only describing one quantity of interest (concentration, temperature or velocity) was −2, the point is that the same approach can be used to solve DCMB, DEB and N-S equations (however, examples don't need to include more than one quantity). No general answer given (just examples) −4 or less.

2. (30) Given that the DCMB equation in cylindrical coordinates is:

$$\frac{\partial C_A}{\partial t} + \overset{1}{\cancel{u_r \frac{\partial C_A}{\partial r}}} + \overset{1}{\cancel{\frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta}}} + \overset{1}{\cancel{u_z \frac{\partial C_A}{\partial z}}} = D_{AB} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C_A}{\partial r} \right) + \overset{2}{\cancel{\frac{1}{r^2} \frac{\partial^2 C_A}{\partial \theta^2}}} + \frac{\partial^2 C_A}{\partial z^2} \right) + r_A$$

Cross out the irrelevant terms and write the initial and boundary conditions necessary to find the transient oxygen concentration distribution in a vascular scaffold implant. Assume the scaffold is a solid cylinder of radius, R, and length, L. Assume R and L are comparable such that it would be inappropriate to assume infinite length. Call the initial oxygen concentration distribution in the implant $C_i(r, l)$. Assume that at the time of implantation the oxygen concentration at the surface of the implant instantaneously changes to C_s (what type of BC is this?). Assume a metabolic consumption rate of oxygen throughout the implant (by pre-seeded cells) of bC, where b is a constant and C is the local oxygen concentration. Supply other necessary assumptions, if any.

Assumptions: 1) Solid; 2) Circumferential symmetry. (2 pts, each term left in or incorrectly dropped)

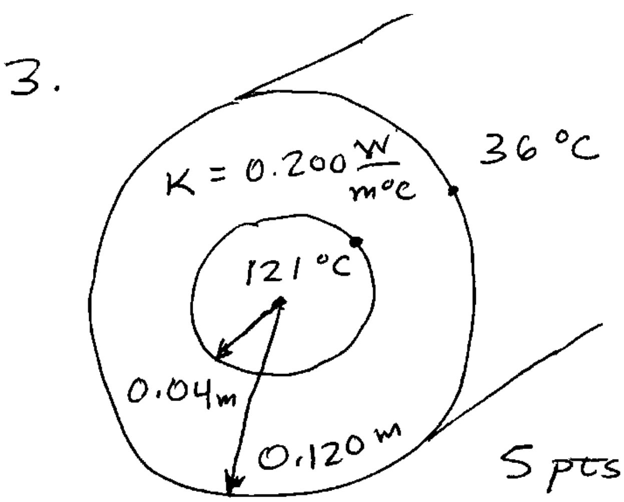
I.C.: $C_i = C_i(r, l)$ (given) (6 pts)

B.C.: $C = C_s$ @ $r = R$, $-L/2 < l < L/2$ and $L = -L/2$ & $L/2$, $0 < r < R$ (this is a Dirichlet type B.C. (1 pts) and was given) also $\partial C/\partial z = 0$ and $l = 0$ and $\partial C/\partial r = 0$ @ $r = 0$ by symmetry (6 pts)

Assume D_A is constant (2 pts); $r_A = bC$ (given) (5 pts)

3. (30) Say that as part of a "Green" audit of the hospital where you find work after graduation, you're asked to estimate the rate a heat loss (transfer) through the insulation on a cylindrical steam pipe feeding an autoclave. Assume the inner surface of the insulation is at 121°C and that you measure the temperature of the outer surface using a non-contact IR thermometer and find it to be 36°C. Assume the insulation has an inner radius of 0.040 m and an outer radius of 0.120 cm and to have a thermal conductivity of 0.200 W/ m·°C. Find the rate of heat transfer per meter of length.

3.



5 pts

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi L K} \quad (17\text{-}39)$$

5 PTS

$$\dot{Q}_{cyl} = \frac{T_2 - T_1}{R_{cyl}} \quad (17\text{-}38)$$

5 PTS

This is a case of s.s. conduction in a cylinder so Eq. 17-38 + 17-39 apply.

$$\dot{Q}/L = \frac{2\pi K}{\ln(r_2/r_1)}(T_2 - T_1)$$

5 PTS

$$= \frac{2\pi\left(0.200 \frac{W}{m\,\cancel{°C}}\right)}{\ln\left(\frac{0.120\,\cancel{m}}{0.040\,\cancel{m}}\right)}(121 - 36\,\cancel{°C})$$

$$= \left(1.14 \frac{W}{m\,°C}\right)(85\,°C) = \underline{\underline{97.2\ W/m}}$$

10 PTS

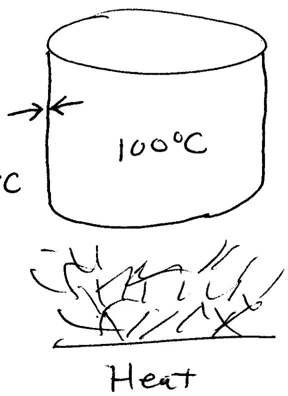
4. (30) Water is being boiled on a gas stove in an open aluminum pan. The effective area for heat transfer is 0.0350 m^2. Assume the water keeps the inner surface of the pan at 100°C. Assume the hot gases surrounding the outside of the pan are at 600°C with the convective coefficient, *h*, being 75.0 $W/m^2\cdot°C$. The pan wall is 0.00100 m thick and has a thermal conductivity, *k*, of 240 W/ m·°C. Assume a flat planar geometry and that the latent heat of vaporization of water is 2256.5 kJ/kg. Find the temperature of the outer surface of the pan and the rate of evaporation (in kg/s) of the water.

4. Assume planar geometry

3 pts

$L = 0.001\ m$

$K = 240\ W/m°C$



$h = 75.0\ W/m^2°C$

$A = 0.0350\ m^2$

600°C

$$\left.\begin{aligned}\Delta H_{vap} &= 2256.5\ kJ/kg\\ &= 2256.5\ J/g\end{aligned}\right\}\ \text{given}$$

100°C —R_{wall}— $T = ?$ —$R_{conv.}$— 600°C 5 pts

3 pts $$R_{wall} = \frac{L}{KA} = \frac{0.00100\ \cancel{m}}{(240\ \frac{W}{\cancel{m}°C})(0.0350\ \cancel{m^2})} = 1.19 \times 10^{-4}\ \frac{°C}{W}$$

3 pts $$R_{conv} = \frac{1}{hA} = \frac{1}{(75\ W/m^2°C)(0.0350\ m^2)} = 0.381\ \frac{°C}{W}$$

$$\dot{Q} = \frac{\Delta T}{R_{total}} = \frac{600 - 100\ °C}{1.19 \times 10^{-4} + 0.381\ °C/W} = \underline{1310\ W}$$ 3 pts

$$\dot{m} = \frac{\dot{Q}}{\Delta H} = \frac{1310\ \cancel{J}/s}{2256.5\ \cancel{J}/g} = \underline{0.581\ g/s}$$ 2 pts

$$\Delta T = \dot{Q}R = (1310\ W)(1.19 \times 10^{-4}\ °C/W) = \underline{0.156}\ °C$$

3 pts

$$T = T_{water} + \Delta T = 100 + 0.156 = \underline{100.2}\ °C$$