Notation

$E[f(X,Y)]$ ← The mean of $f(X,Y)$

e.g.

$E[X+Y] \equiv$ the mean of $X+Y$

suppose $Z = X+Y$

~~to~~ let m_Z = the mean of $Z = E[Z]$

and similarly for m_X & m_Y.

then $m_Z = m_X + m_Y$

~~to~~ So we can say...

$$E[X+Y] = m_X + m_Y$$

$Var(f(X,Y))$ ← The variance of $f(X,Y)$

e.g.

$Var(X) \equiv$ the variance of X

suppose X has a std. dev. of σ_X

then we know that the variance is by definition the square of the std. dev., so

$$Var(X) = \sigma_X^2$$

we can also define variance like this:

$$Var(X) \equiv E[(X - E[X])^2] = E[(X - m_X)^2]$$

that is, the mean of the square of the difference from the random values from their mean

"Mean Square Error"

One sample

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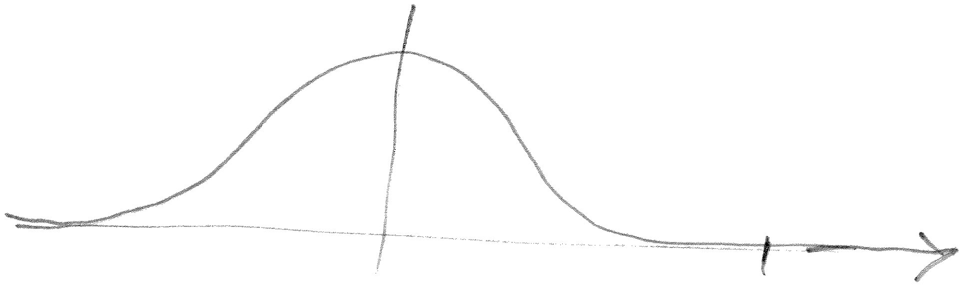
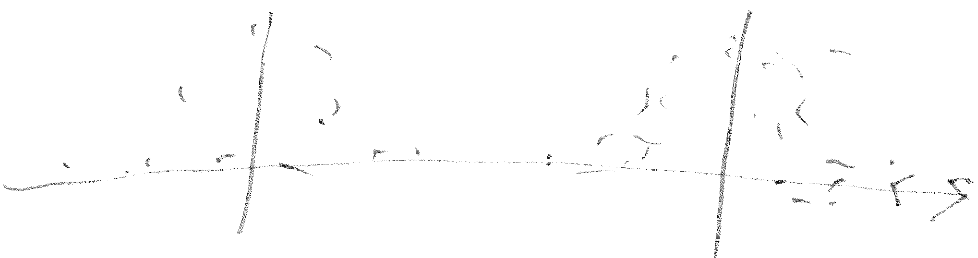
(and 9-2)



Means the same?



Need statistic-of-sameness

Difference of means:

$$D = M_2 - M_1 = \frac{X_{2,1} + X_{2,2} + \dots X_{2,N}}{N} - \frac{X_{1,1} + X_{1,2} + X_{1,3} + X_{1,N}}{N}$$

Assume samples are distributed according to $\sim N(m, \sigma)$

normal (Gaussian) · mean · known · std

(see next page)

$$D \sim N\left(\underbrace{0}_{\text{mean}}, \underbrace{\sqrt{\frac{\sigma^2 + \sigma^2 + \sigma^2 + \dots}{N^2} + \frac{\sigma^2 + \dots \sigma^2}{N^2}}}_{\text{std}}\right)$$

normal (Gaussian distribution)

$$= N\left(0, \sqrt{\sigma^2 + \sigma^2}\right) = N\left(0, \sigma\frac{\sqrt{2}}{\sqrt{N}}\right)$$

so $\frac{\sqrt{N} D}{\sqrt{2}\sigma} \sim N(0,1)$ ← charts for this.

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$\cancel{VAR((X - E[X])^2}$

$\cancel{VAR}$ $Var(X) = E[(X-E[X])^2]$

$Var(aX) = E[(aX - E[aX])^2] = a^2 Var(X)$

$Var(D) = Var\left(\cancel{\sigma^2+\sigma}\ \frac{X_{2,1}+X_{2,2}+\ldots+X_{2,N}}{N} - \frac{X_{1,1}+X_{1,2}+\ldots}{N}\right)$

$= Var\left(\frac{X_{2,1}}{N}\right) + Var\left(\frac{X_{2,2}}{N}\right) + \ldots + Var\left(\frac{-X_{1,1}}{N}\right) + Var\left(\frac{-X_{1,2}}{N}\right)$

$= \frac{1}{N^2} Var(X_{2,1}) + \ldots + \frac{(-1)^2}{N^2} Var(X_{1,1}) + \frac{(-1)^2}{N^2} Var(X_{1,2})$

$= N \cdot \left(\frac{1}{N^2}\sigma^2\right) + N \cdot \left(\frac{1}{N^2}\sigma^2\right)$

$= \frac{2\sigma^2}{N}$

$\sigma_D = std(D) = \sqrt{Var(D)} = \sqrt{\frac{2}{N}}\,\sigma$

so $m_D = 0$, $\sigma_D = \sqrt{\frac{2}{N}}\,\sigma$

Let $Z = \frac{D}{\sqrt{\frac{2}{N}}} = \sqrt{\frac{N}{2}}\,\frac{D}{\cancel{\sigma}}$

$m_Z = 0$ $\quad \cancel{\sigma_D =}\ \sigma_Z = \frac{\sqrt{\frac{N}{2}}\,\sigma_D}{\sigma} = \sqrt{\frac{N}{2}}\sqrt{\frac{2}{N}}\,\frac{\sigma}{\sigma} = \cancel{\sigma} = 1.$

so Z has a mean of 0 and ~~the same std. as the samples~~ a std. of 1.